MACHINE DESIGN II
MEC 3110

BY

PROF. M. NAUSHAD ALAM

MECHANICAL ENGINEERING DEPT.

A.M.U. ALIGARH

BASICS OF MACHINE DESIGN

- **Machine Design :**

  It is defined as the use of scientific principles,
  Technical informations,
  and imagination
  in the description of a machine or mechanical system,
  to perform specific functions with maximum economy and efficiency.

- Design is an innovative and highly iterative process.

- A designer uses principles of basic engineering sciences such as physics, mathematics, statics, dynamics, thermodynamics, heat transfer, fluid mechanics, Vibration etc.

Course Objectives

1. Understanding the process and methods of design of machines elements.
2. Abilities of developing equations pertaining to the design of machines.
3. Knowledge of different materials and their properties for designing the components of machine elements and the ability to design new machines or modify existing machine according to the need.

Course Outcomes

After taking this course students should be able to,

1. Design and analyse different types of clutches, brakes and welded connections.

2. Select and design appropriate bearing as per the requirement.

3. Identify different types of springs and design the same as per the requirement.

4. Design and analyze geared transmission system.

Syllabus

Unit-I

Welded Joints: Types of Welded connections, Design of Simple and eccentrically loaded welded connections. Clutches & Brakes: Plate, Cone and Centrifugal Clutches, Classification and Design of Brakes.

Unit-II

Bearings & Lubrication: Laws of friction, Lubrication, Hydrodynamic and Hydrostatic bearings, Ball and Roller bearings, Method of load estimation and Selection of bearings.

Unit-III

Springs: Design of helical springs, design of torsion and leaf springs, elementary idea of rubber springs.

Unit-IV

Power Transmission with Toothed Gears: Selection of Gears and Gear Materials, Tooth Forces, Design of different types of Gears.

BOOKS

1. **Joseph E. Shigley;** Mechanical Engineering Design, McGraw Hill.

2. **V. B. Bhandari;** Design of Machine Elements, Tata McGraw-Hill Education

3. **M.F. Spott;** Design of Machine Element, Prentice Hall.

4. **Design Data Handbook** for Mechanical Engineers, K. Mahadevan and K. Balaveera Reddy, CBS Pub.

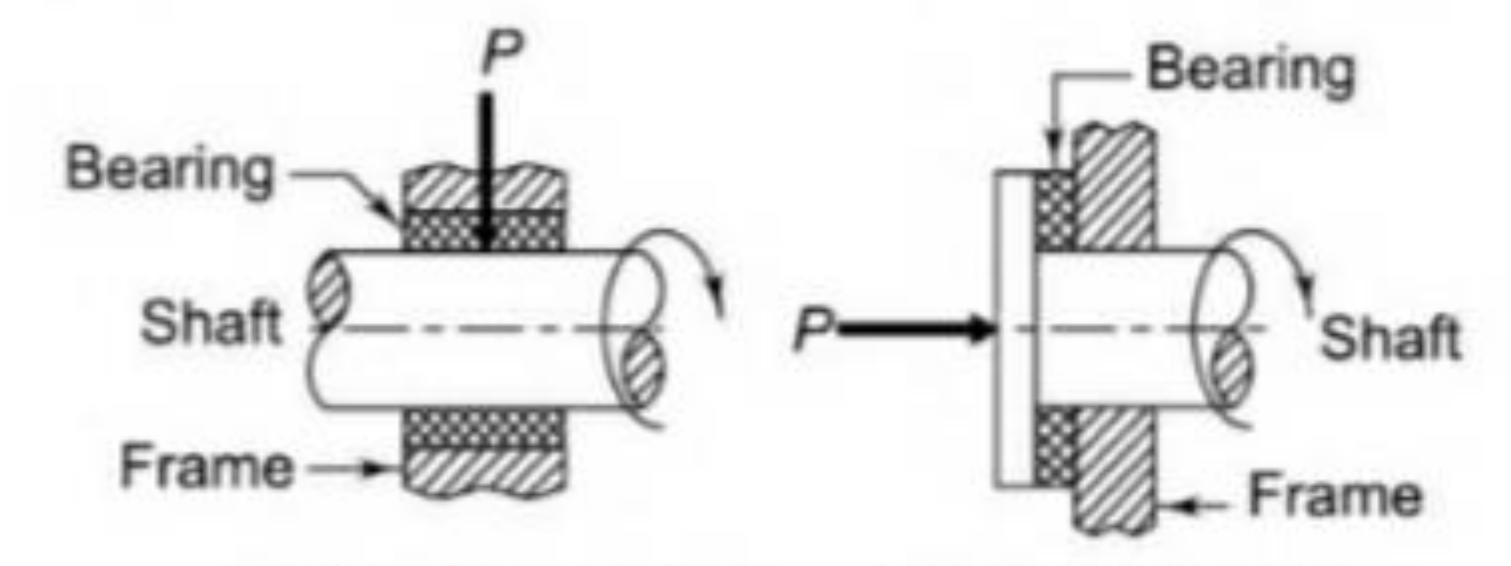
5. **Standard Handbook of Machine Design,** Joseph E Shigley, and Charles R. Mischke, McGraw-Hill Pub.

BEARINGS

- Bearings are machine elements which are used to support a rotating member viz., a shaft.
- They transmit the load from a rotating member to a stationary member known as frame or housing.
- They allow relative motion between two members in one or two directions with minimum friction, and also prevent the motion in the direction of the applied load.
- Bearings support a shaft or an axle and holds them in correct positions.

Classification of Bearings

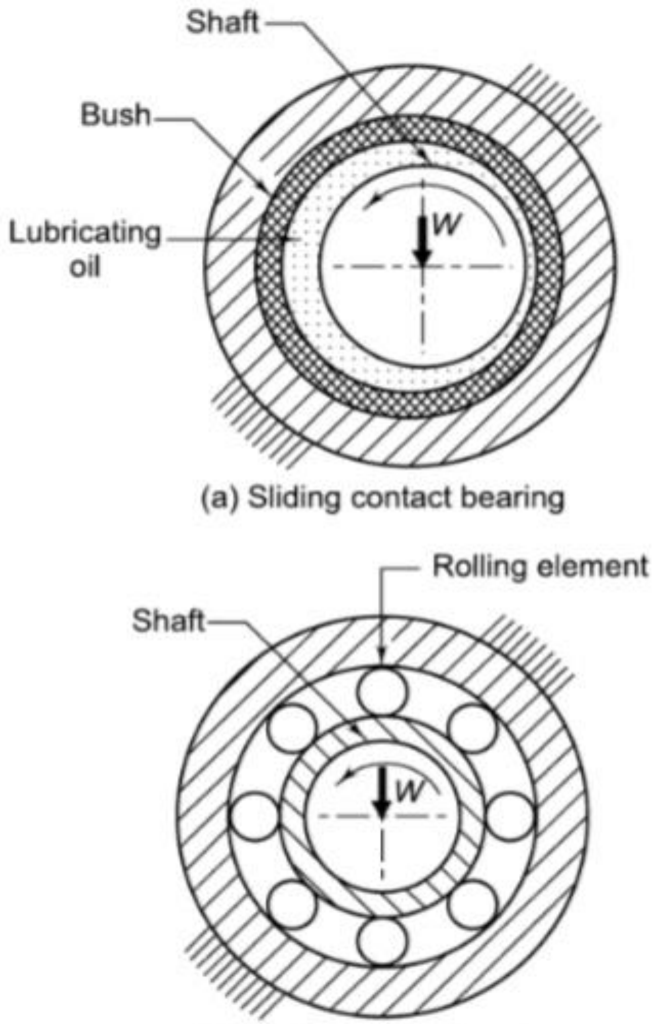
- According to direction of force;
  1. Radial Bearing : Supports the load which is perpendicular to axis of the shaft,
  2. Thrust Bearing : Supports the load which acts along the axis of the shaft.



(a) Radial bearing (b) Thrust bearing

Classification of Bearings

- According to type of friction or contact :
  1. Sliding Contact (Journal)
  2. Rolling Contact (Antifriction)



(a) Sliding contact bearing

(b) Rolling contact bearing

Applications of Bearings

- Sliding contact
  - Crankshaft bearings in petrol and diesel engines,
  - Centrifugal pumps
  - Large size electric motors
  - Steam and gas Turbines,concrete mixture, Rope conveyor and marine installations
- Rolling contact
  - Automobile front and rear axles, machine tool spindles, Gear boxes.

Lubrication

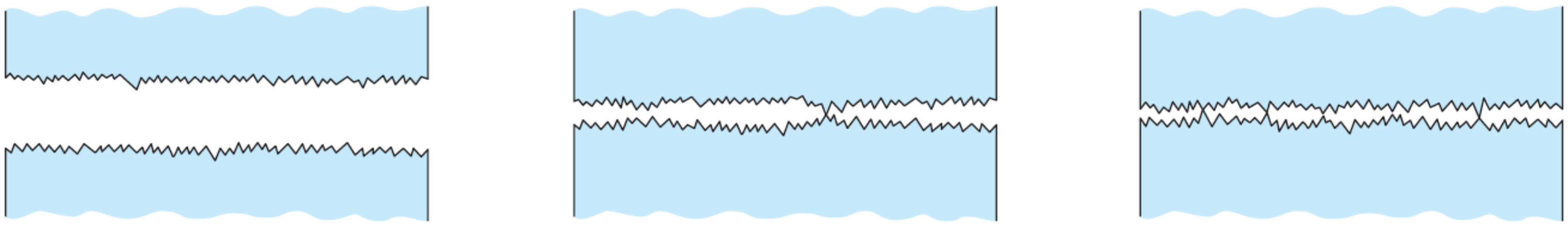
- The objective of lubrication is to reduce friction wear and heating of machine parts that move relative to each other.
- A Lubricant is a substance that when inserted between the moving surfaces, accomplishes theses purposes.
- Lubricants are usually liquid but can be a solid, such as graphite, or molybdenum disulfide, or a gas, such as pressurized air.

Types of lubrication

1. **Thick film lubrication or H**drodynamic lubrication or : the surfaces are separated by thick film of lubricant and there is no metal-to-metal contact. The film thickness is anywhere from 8 to 20 μm. Hydrodynamic lubrication.

2. **Thin film lubrication –** Here even though the surfaces are separated by thin film of lubricant, at some high spots Metal-to-metal contact does exist, it also known as mixed film lubrication. Surface wear is mild. The coefficient of friction commonly ranges from 0.004 to 0.10.

3. **Boundary lubrication –** Here the surface contact is continuous and lubricant is continuously smeared over the surfaces and provides a continuously renewed adsorbed surface film which reduces the friction and wear. The typical coefficient of friction is 0.05 to 0.20.



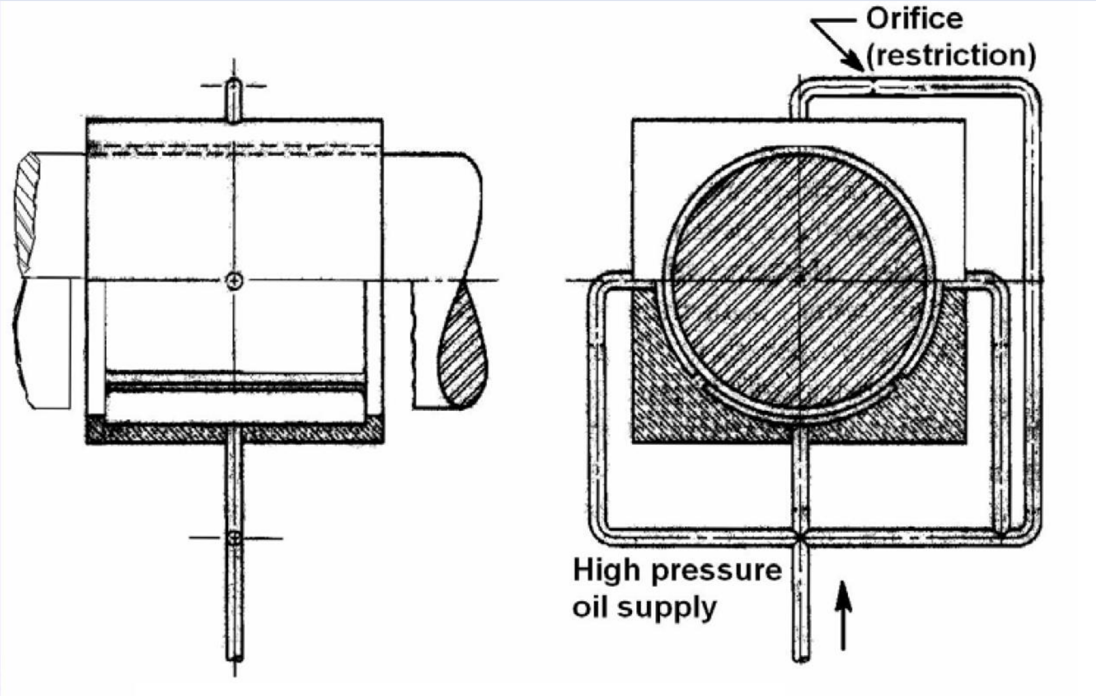
(*a*) Hydrodynamic (surface separated)

(*b*) Mixed film (intermittent local contact)

(*c*) Boundary (continuous and extensive local contact)

Types of lubrication

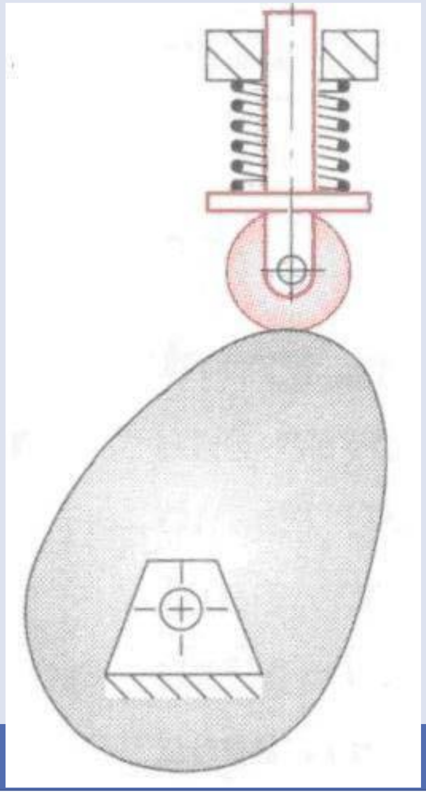
4. **Hydrostatic lubrication** :
   - In these bearings, externally pressurized lubricant is fed into the bearings to separate the surfaces with thick film of lubricant.
   - These types of bearings do not require the motion of the surfaces to generate the lubricant film. Hence they can operate from very low speed to high speed.



Types of lubrication

5. Elastohydrodynamic lubrication:

- Rolling contact bearings come under this category. The oil film thickness is very small.
- The contact pressures are going to be very high. Hence to prevent the metal-to-metal contact, surface finishes are to be of high quality.
- Such a type of lubrication can be seen in
  - Gears,
  - Rolling contact bearings,
  - Cams etc.



Types of lubrication

6. **Solid film lubrication:**

For extreme temperature operations ordinary mineral oils are not satisfactory. Few examples of Sold lubricants are:

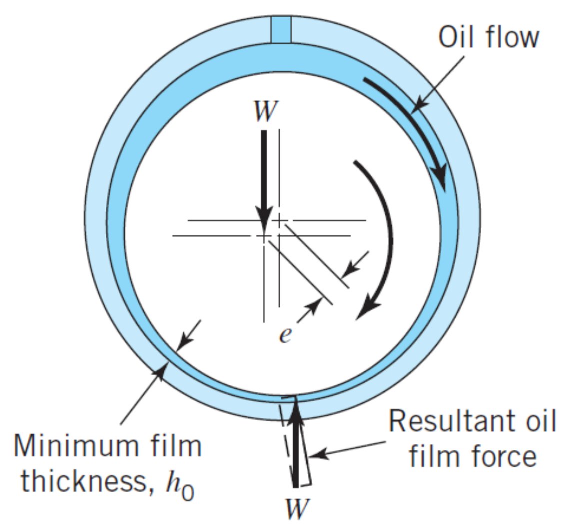
- Graphite,
- Molybdenum disulphide
- and their combination,

Applications :

- Furnace applications
- Hot drawing mills
- Trunion bearings of liquid metal handling systems

Hydrodynamic lubrication

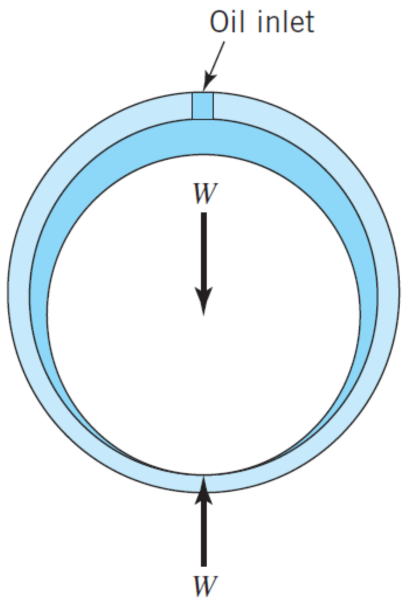
- Also called *full-film,* or *fluid lubrication has following characteristics:*
- The load-carrying surfaces of the bearing are separated by a thick film of lubricant.
- Requires an adequate supply of lubricant at all times.
- Does not require introduction of the lubricant under pressure.



Hydrodynamic Lubrication

Hydrodynamic lubrication

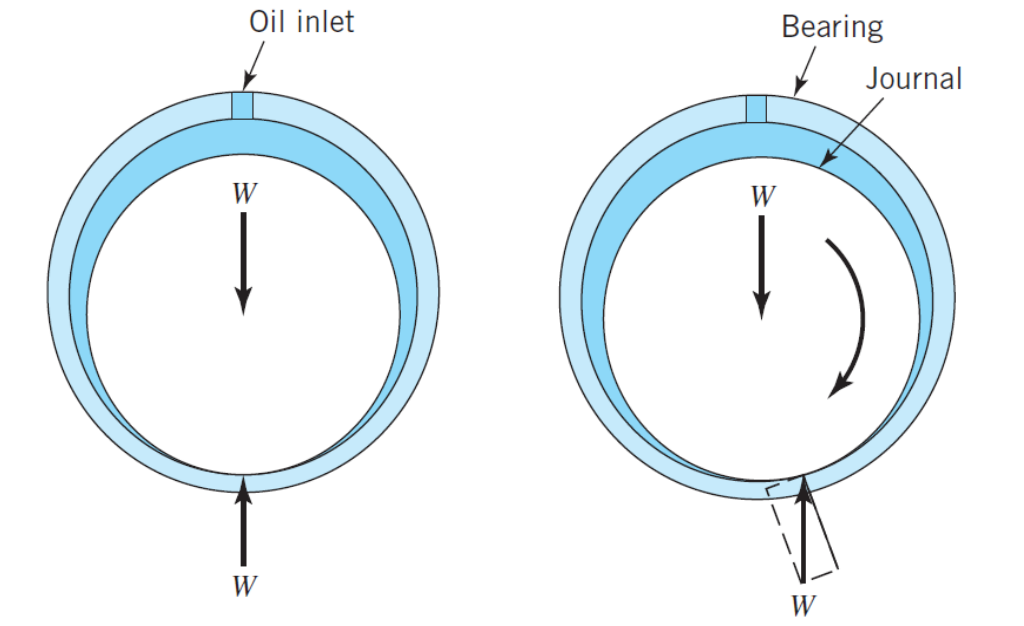
- Initially the loaded journal bearing is at rest.
- The bearing clearance space is filled with oil.
- The load (*W*) has squeezed out the oil film at the bottom.



Journal at Rest

Hydrodynamic lubrication

- The shaft starts Slow clockwise rotation.
- This will cause it to roll to the right position as it tries to "climb the wall" of the bearing surface. .
- Continuous *slow* rotation would cause the shaft to stay in this position.

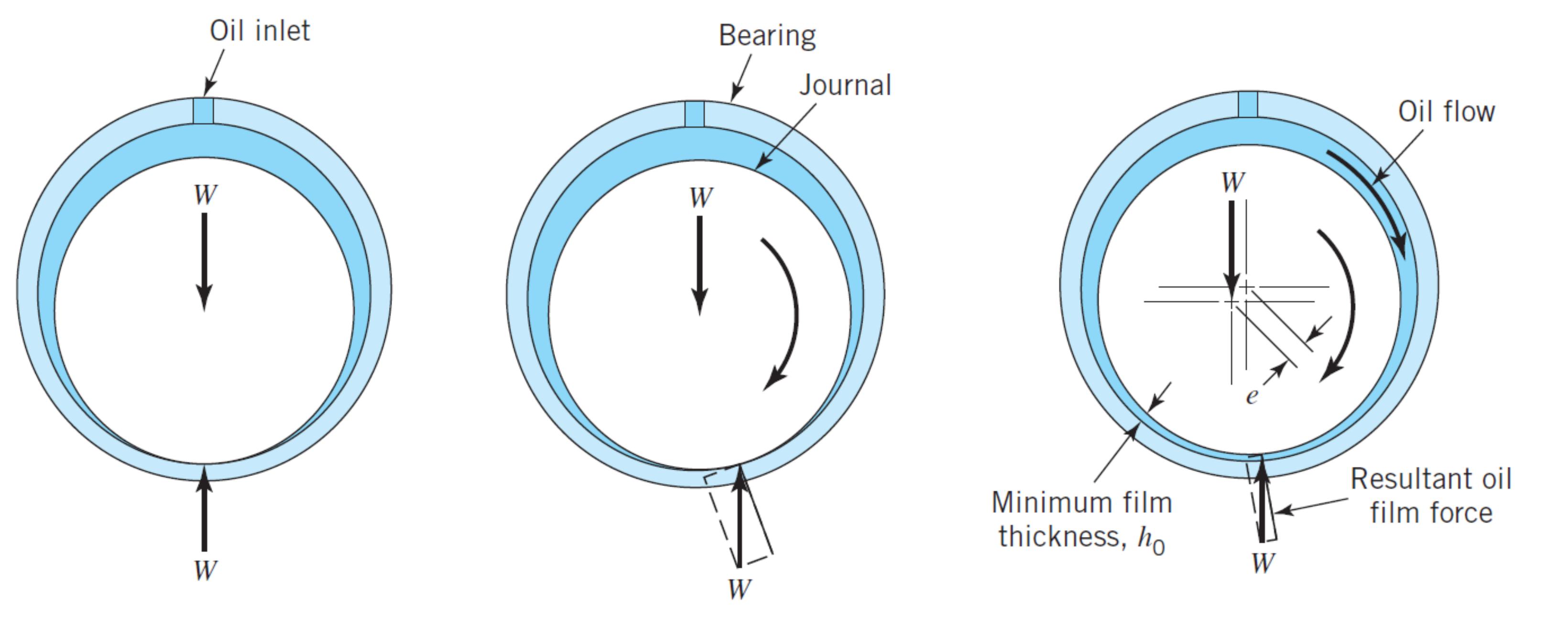


Journal at Rest **Boundary Lubriaction**

The result is the boundary lubrication.

Hydrodynamic lubrication

- As the shaft rotating speed is increased, more and more oil goes into the contact zone.
- Finally a speed is reached at which the pressure built up in the contact zone is high enough to "float" the shaft.
- Under suitable conditions full separation of the journal and bearing surfaces occurs.



Journal at Rest **Boundary Lubriaction** **Hydrodynamic Lubrication**

MACHINE DESIGN II
MEC 3110

Lecture-2

BY

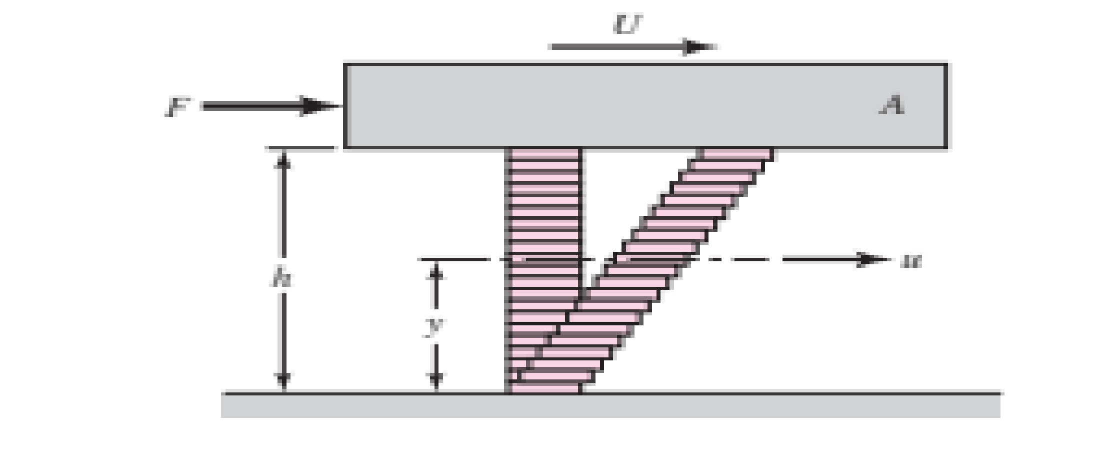
PROF. M. NAUSHAD ALAM

MECHANICAL ENGINEERING DEPT.
A.M.U. ALIGARH

VISCOCITY

It is defined as the internal frictional resistance offered by a fluid to change its shape or relative motion of its parts

- In Fig. let a plate *A* be moving with a velocity *U* on a film of lubricant of thickness *h*.The film as composed of a series of horizontal layers and the force *F* causing
- these layers to deform or slide on one another just like a deck of cards. The layers in contact
- with the moving plate are assumed to have a velocity *U*;
- Newton's viscous effect states that the shear stress in the fluid is proportional to the rate of change of velocity with respect to *y*



Newtons Viscous Effect

- Newton's viscous effect states that the shear stress in the fluid is proportional to the rate of change of velocity with respect to *y*.

$$\tau = \frac{F}{A} = \mu \frac{du}{dy}$$

- where μ is the constant of proportionality and defines absolute viscosity, also called dynamic viscosity.
- The derivative du/dy is the rate of change of velocity with distance and may be called the rate of shear, or the velocity gradient.
- The viscosity μ is thus a measure of the internal frictional resistance of the fluid. For most lubricating fluids, the rate of shear is constant, and du/dy = U/h. Thus, from previous equation

$$\tau = \frac{F}{A} = \mu \frac{U}{h}$$

- Unit of dynamic viscosity *poise* (P) in CGS,
- Unit of kinematic viscosity called the *stoke* (St). per square centimeter (dyn · s/cm2).
- It has been customary to use the centipoise (cP) in analysis, because its value is more convenient.
- When the viscosity is expressed in centipoises, it is designated by *Z*.
- The conversion from cgs units to SI and ips units is as follows:

$$1 \text{ reyn} = \frac{1 \text{ lb} \cdot \text{s}}{\text{in.}^2} = \frac{6890 \text{ N} \cdot \text{s}}{\text{m}^2} = 6890 \text{ Pa} \cdot \text{s}$$

$$(1 \text{ cp} = 1 \text{ mPa} \cdot \text{s}).$$

SAYBOLT VISCOMETER

- The ASTM standard method for determining viscosity uses an instrument called the Saybolt Universal Viscosimeter.
- The method consists of measuring the time in seconds for 60 mL of lubricant at a specified temperature to run through a tube 17.6 mm in diameter and 12.25 mm long.

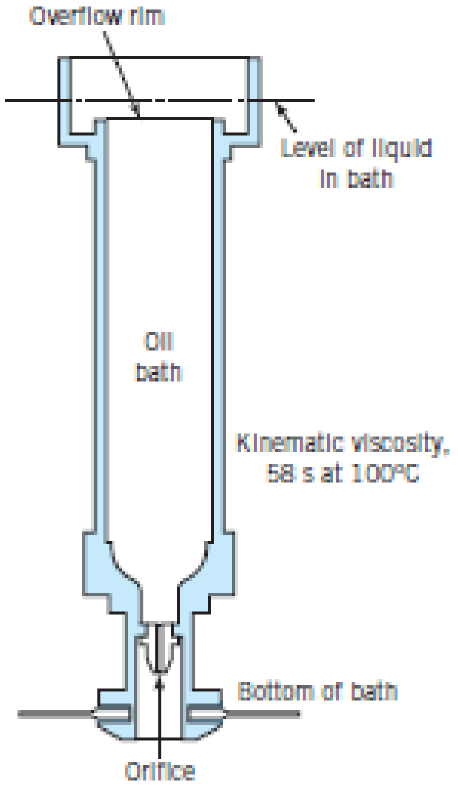


FIGURE 13.7
Saybolt viscometer.

- The result is called the *kinematic viscosity,* and in the past the unit of the square centimeter per second has been used. One square centimeter per second is defined as a *stoke*.

- By the use of the *Hagen-Poiseuille law,* the kinematic viscosity based upon seconds Saybolt, also called *Saybolt Universal viscosity* (SUV) in seconds, is

$$Z_k = \left(0.22t - \frac{180}{t}\right)$$

$$\nu(\mathrm{m^2/s}) = 10^{-6} Z_k \ (\mathrm{cSt})$$

$$\nu = \left(0.22t - \frac{180}{t}\right)(10^{-6})$$

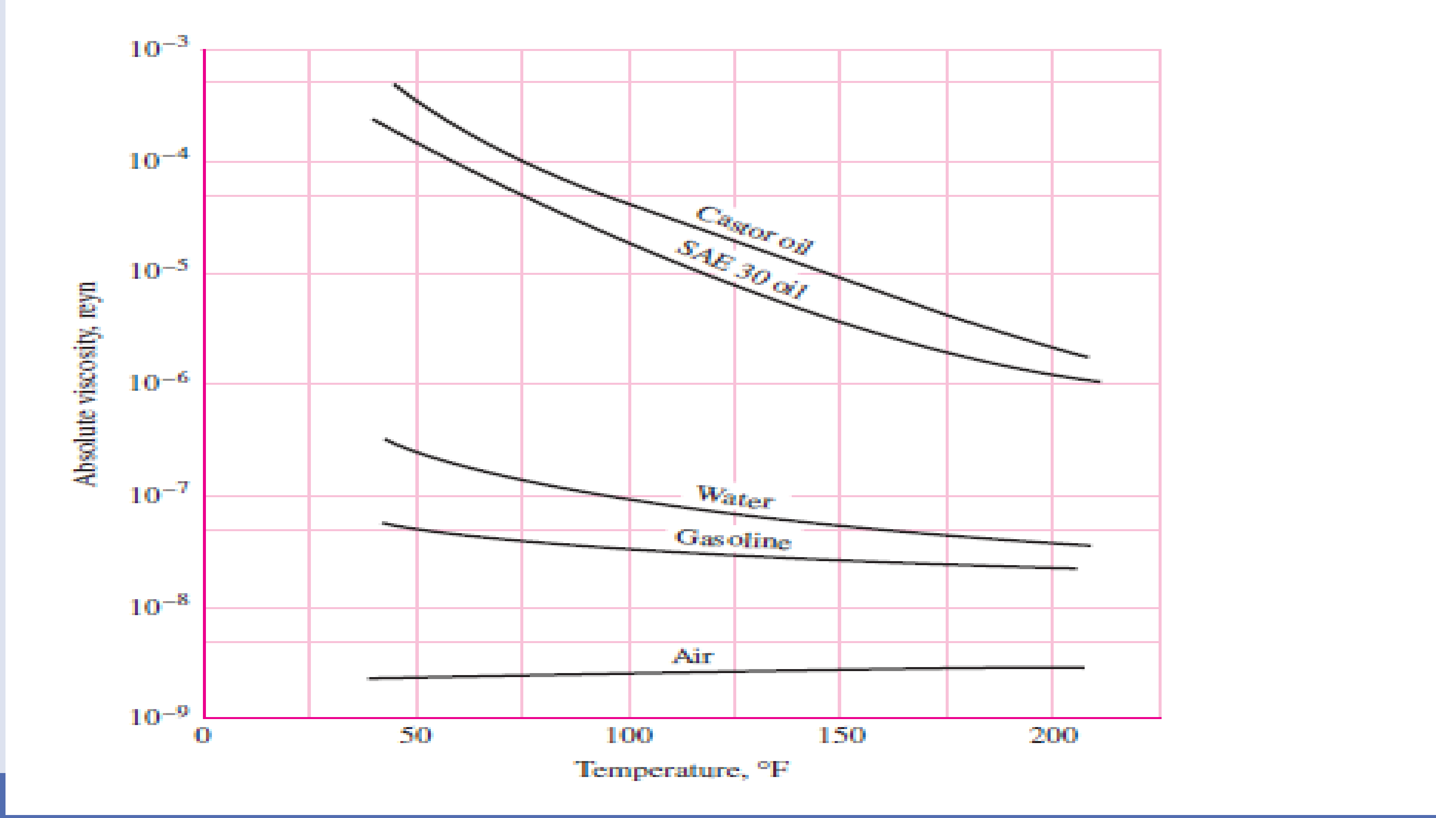
$$\nu(\mathrm{m^2/s}) = 10^{-6} Z_k \ (\mathrm{cSt})$$

$$\nu = \left(0.22t - \frac{180}{t}\right) (10^{-6})$$

$$\mu = \rho \left(0.22t - \frac{180}{t}\right) (10^{-6})$$

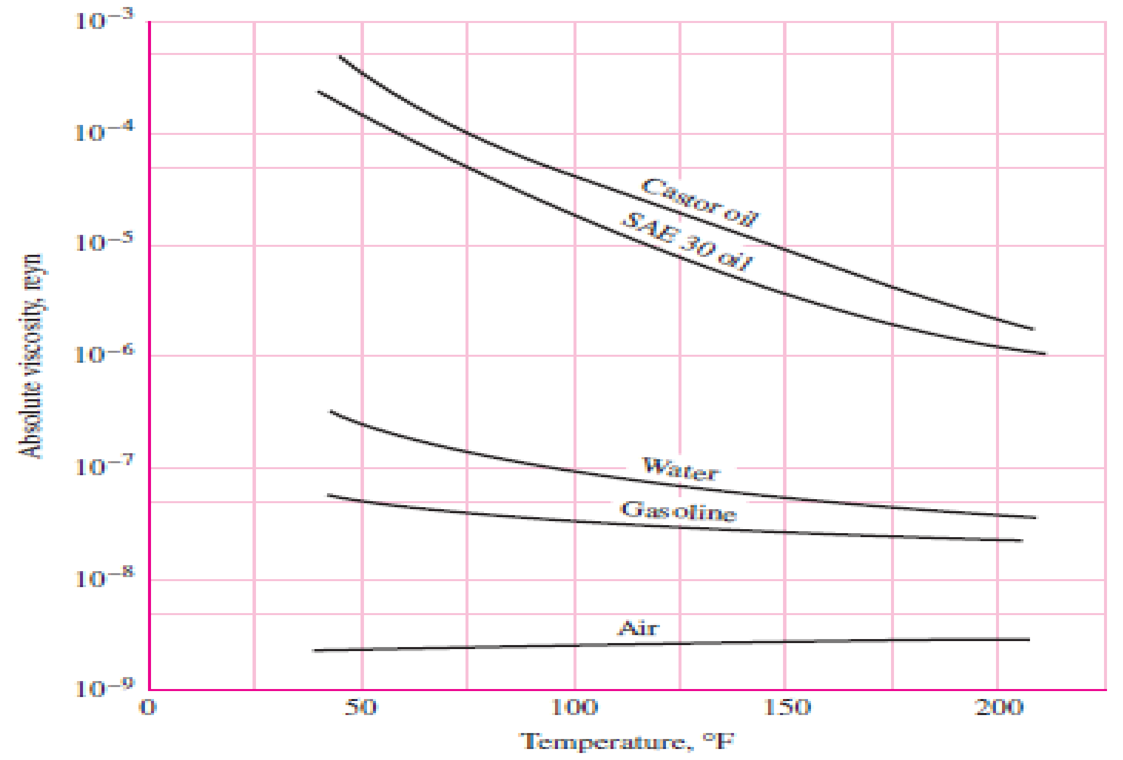
Temperature and Pressure Effects on Viscosity

- Multigrade oils, as SAE 10W-40, have less variation of viscosity with temperature than straight-run petroleum oils that have a single-grade designation (as SAE 40 or SAE 10W).



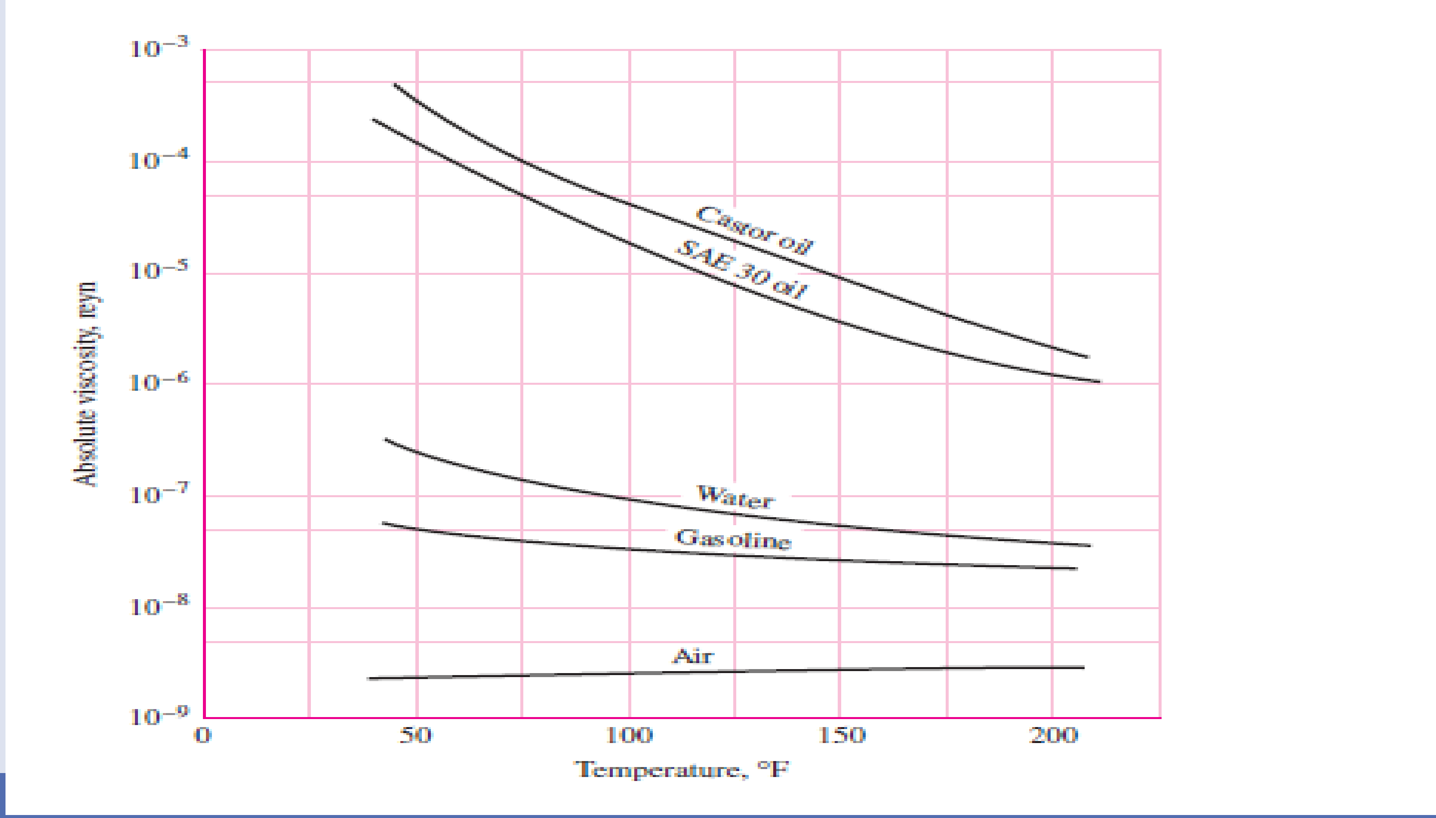
Temperature and Pressure Effects on Viscosity

- The measure of variation in viscosity with temperature is the *viscosity index* (abbreviated *VI*). oils, for example, have relatively little variation of viscosity with temperature.



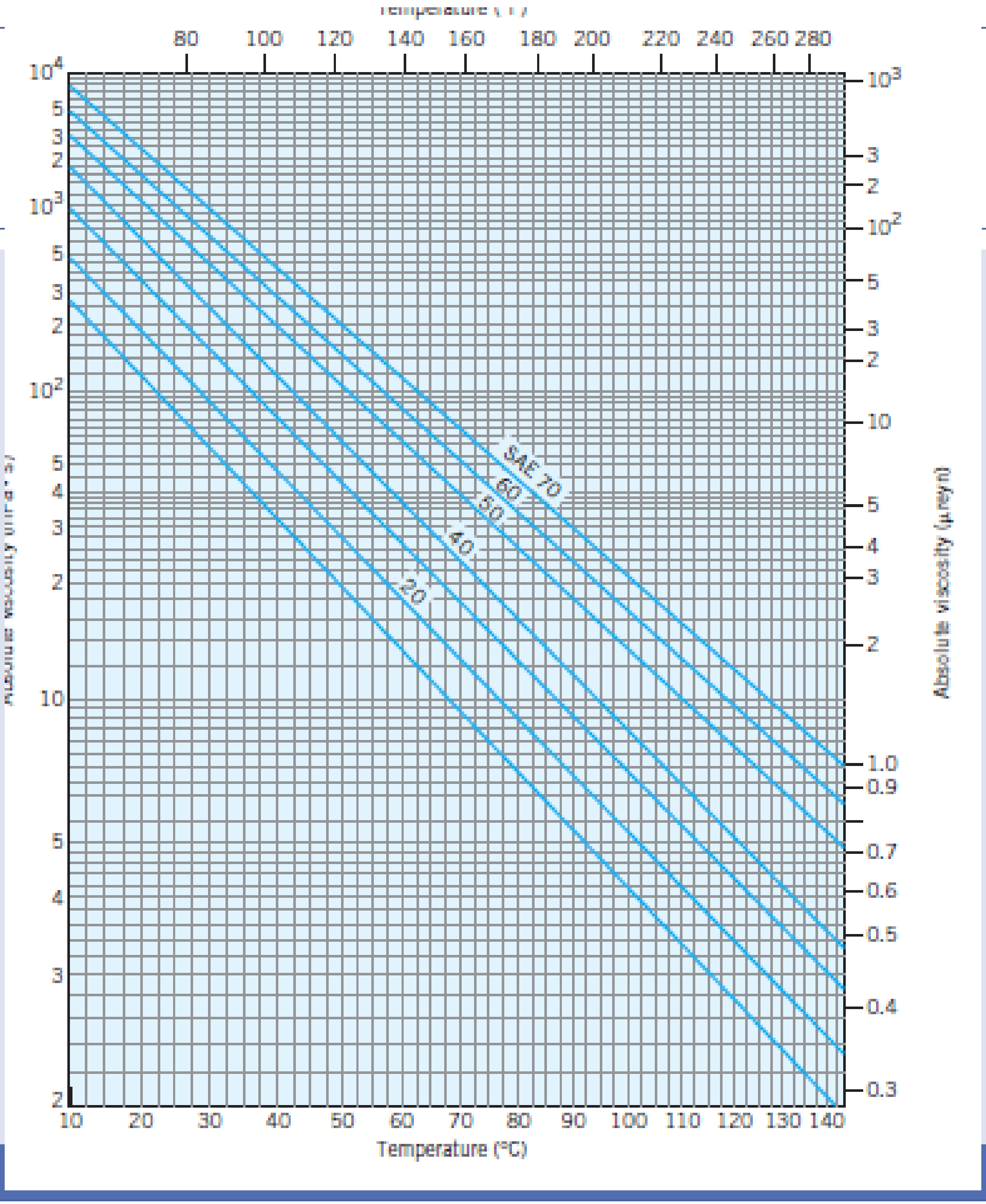
Temperature and Pressure Effects on Viscosity

- The viscosity index of petroleum oils can be increased, as in the production of multigrade oils, by the use of viscosity index improvers (additives).
- All lubricating oils experience an increase in viscosity with pressure.



Temperature vs Viscocity

.



$$\upsilon = \text{Kinematic viscosity} = \frac{\text{absolute viscosity}}{\text{mass density}} \tag{13.3}$$

Units are length^2/time, as cm^2/s, which is named the *stoke*, abbreviated St.

Absolute viscosities can be obtained from Saybolt viscometer measurements (time S, in seconds) by the equations

$$\mu(\text{mPa}\cdot\text{s, or cp}) = \left(0.22S - \frac{180}{S}\right)\rho \tag{13.4}$$

and

$$\mu(\mu\text{reyn}) = 0.145\left(0.22S - \frac{180}{S}\right)\rho \tag{13.5}$$

where ρ is the mass density in grams per cubic centimeter (which is numerically equal to the specific gravity). For petroleum oils the mass density at 60°F (15.6°C) is approximately 0.89 g/cm^3. At other temperatures the mass density is

$$\rho = 0.89 - 0.00063(°\text{C} - 15.6) \tag{13.6a}$$

$$= 0.89 - 0.00035(°\text{F} - 60) \tag{13.6b}$$

Problem -1

- An engine oil has a kinematic viscosity at 100°C corresponding to 58 seconds, as determined from a Saybolt viscometer (Figure).
- What is its corresponding absolute viscosity in millipascal-seconds (or centipoises) and in microreyns?
- To what SAE number does it correspond?

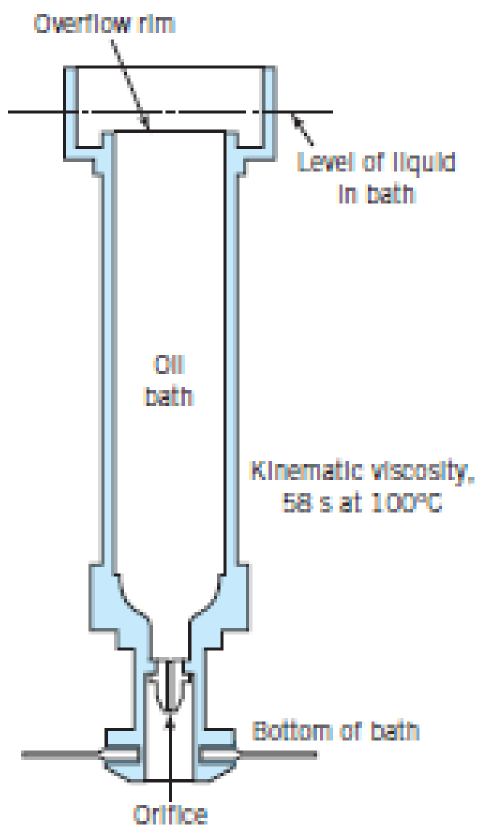


FIGURE 13.7
Saybolt viscometer.

Solution

- **1.** calculating the mass density of the engine oil.
- **2.** calculating absolute viscosities in units of centipoises or microreyns.
- **3.** viscosity–temperature curves for SAE numbered

$$\rho = 0.89 - 0.00063(°C - 15.6)$$
$$= 0.89 - 0.00035(°F - 60)$$

$$\rho = 0.89 - 0.00063(100 - 15.6) = 0.837 \text{ g/cm}^3$$

$$\mu = \left[(0.22)(58) - \frac{180}{58}\right]0.837$$
$$= 8.08 \text{ mPa} \cdot \text{s} \quad (\text{or } 8.08 \text{ cp})$$

$$\mu = 0.145\left[(0.22)(58) - \frac{180}{58}\right]0.837$$
$$= 1.17 \ \mu\text{reyn}$$

From Figure 13.6, the viscosity at 100°C is close to that of an SAE 40 oil.

MACHINE DESIGN II
MEC 3110

Lecture-3

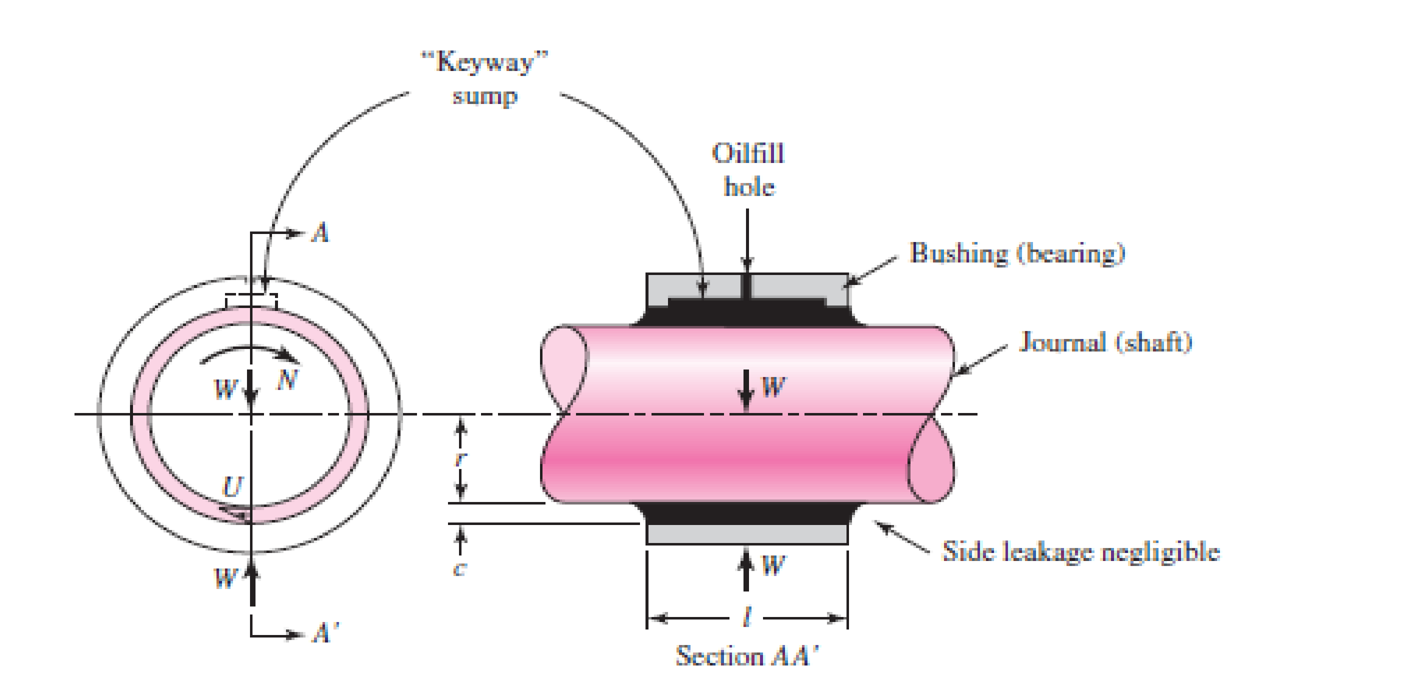
By

PROF. M. NAUSHAD ALAM

MECHANICAL ENGINEERING DEPT.
A.M.U. ALIGARH

Petroff's Equation

- Consider a vertical shaft rotating in a guide bearing.
- It is assumed that the bearing carries a very small load, that the clearance space is completely filled with oil, and that leakage is negligible (Fig.) Denote the radius of the shaft by *r*,



- The radial clearance by c, and the length of the bearing by l, all dimensions being in inches.
- If the shaft rotates at N rev/s, then its surface velocity is $U = 2\pi r N$ in/s.
- Since the shearing stress in the lubricant is equal to the velocity gradient times the viscosity, from Eq. we have
  - F = friction torque/shaft radius = T_f/R
  - $A = 2\pi RL$
  - $U = 2\pi RN$ (where N is in revolutions per *second*)

$$h = c \text{ where } c = \text{radial clearance} = \frac{\text{bearing diameter} - \text{shaft diameter}}{2}$$

$$\tau = \mu \frac{U}{h} = \frac{2\pi r \mu N}{c}$$

- where the radial clearance c has been substituted for the distance h. The force required to shear the film is the stress times the area.
- The torque is the force times the lever arm r. Thus

$$T = (\tau A)(r) = \left(\frac{2\pi r \mu N}{c}\right)(2\pi r l)(r) = \frac{4\pi^2 r^3 l \mu N}{c}$$

…

- If we now designate a small force on the bearing by W, in pounds-force, then the pressure P, in pounds-force per square inch of projected area, is $P = W/2rl$.
- The frictional force is fW, where f is the coefficient of friction, and so the frictional torque is

$$T = fWr = (f)(2rlP)(r) = 2r^2 flP$$

$$f = 2\pi^2 \frac{\mu N}{P} \frac{r}{c}$$

....

$$f = 2\pi^2 \frac{\mu N}{P} \frac{r}{c}$$

- Above Equation is called *Petroff 's equation* and was first published in 1883.
- The two quantities $\mu N/P$ and r/c are very important parameters in lubrication. Substitution of the appropriate dimensions in each parameter will show that they are dimensionless.
- The *bearing characteristic number,* or the *Sommerfeld number,* is defined by the equation

$$S = \left(\frac{r}{c}\right)^2 \frac{\mu N}{P}$$

$$S = \left(\frac{r}{c}\right)^2 \frac{\mu N}{P}$$

- The Sommerfeld number is very important in lubrication analysis because it contains many of the parameters that are specified by the designer.
- Note that it is also dimensionless.
- The quantity r/c is called the *radial clearance ratio*. If we multiply both sides

Problem -2

- A 100-mm-diameter shaft is supported by a bearing of 80-mm length with a diametral clearance of 0.10 mm (Figure). It is lubricated by oil having a viscosity (at the operating temperature) of 50 mPa- s. The shaft rotates 600 rpm and carries a radial load of 5000 N. Estimate the bearing coefficient of friction and power loss using the Petroff approach.

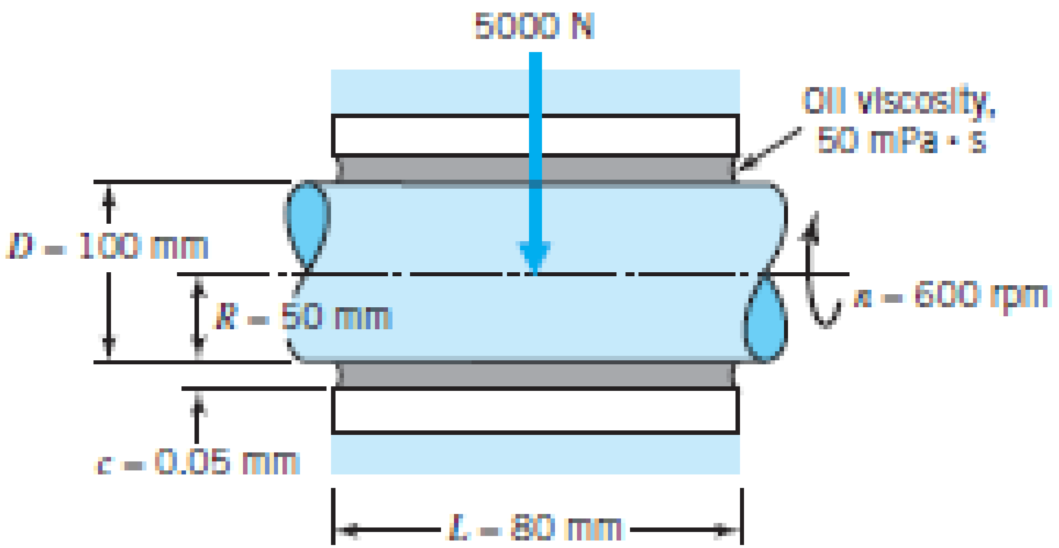


FIGURE 13.9
Journal bearing for Sample Problem 13.2.

Assumptions:

1. There is no eccentricity between the bearing and journal, and no lubricant flow in the axial direction.
2. The frictional drag force is equal to the product of the coefficient of friction times the radial shaft load.

·

Analysis:

1. With the preceding assumptions, Petroff's equation is appropriate. From Eq. 13.7,

$$f = 2\pi^2 \frac{(0.05\ \text{Pa}\cdot\text{s})(10\ \text{rps})}{\dfrac{5000}{0.1 \times 0.08}\text{N/m}^2} \times \frac{50\ \text{mm}}{0.05\ \text{mm}} = 0.0158$$

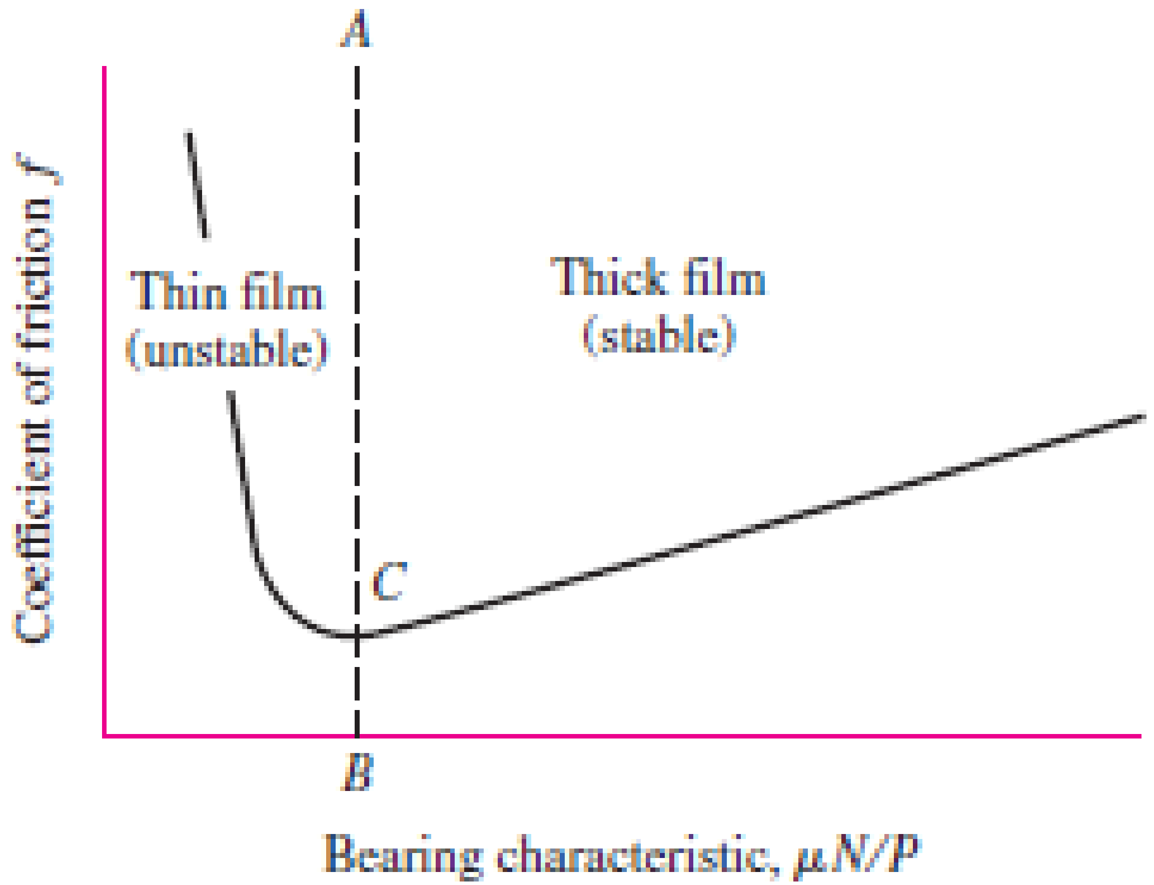
2. Torque friction $T_f = fWD/2 = (0.0158)(5000\ \text{N})(0.1\ \text{m})/2 = 3.95\ \text{N}\cdot\text{m}$

   Note: (1) T_f could also be obtained from Eq. b. (2) The same *calculated* value of T_f would be obtained using any value of W, but the greater the load, the greater the deviation from Petroff's assumption of zero eccentricity.

3. Power $= 2\pi T_f n = 2\pi(3.95\ \text{N}\cdot\text{m})(10\ \text{rps}) = 248\ \text{N}\cdot\text{m/s} = 248\ \text{W}$.

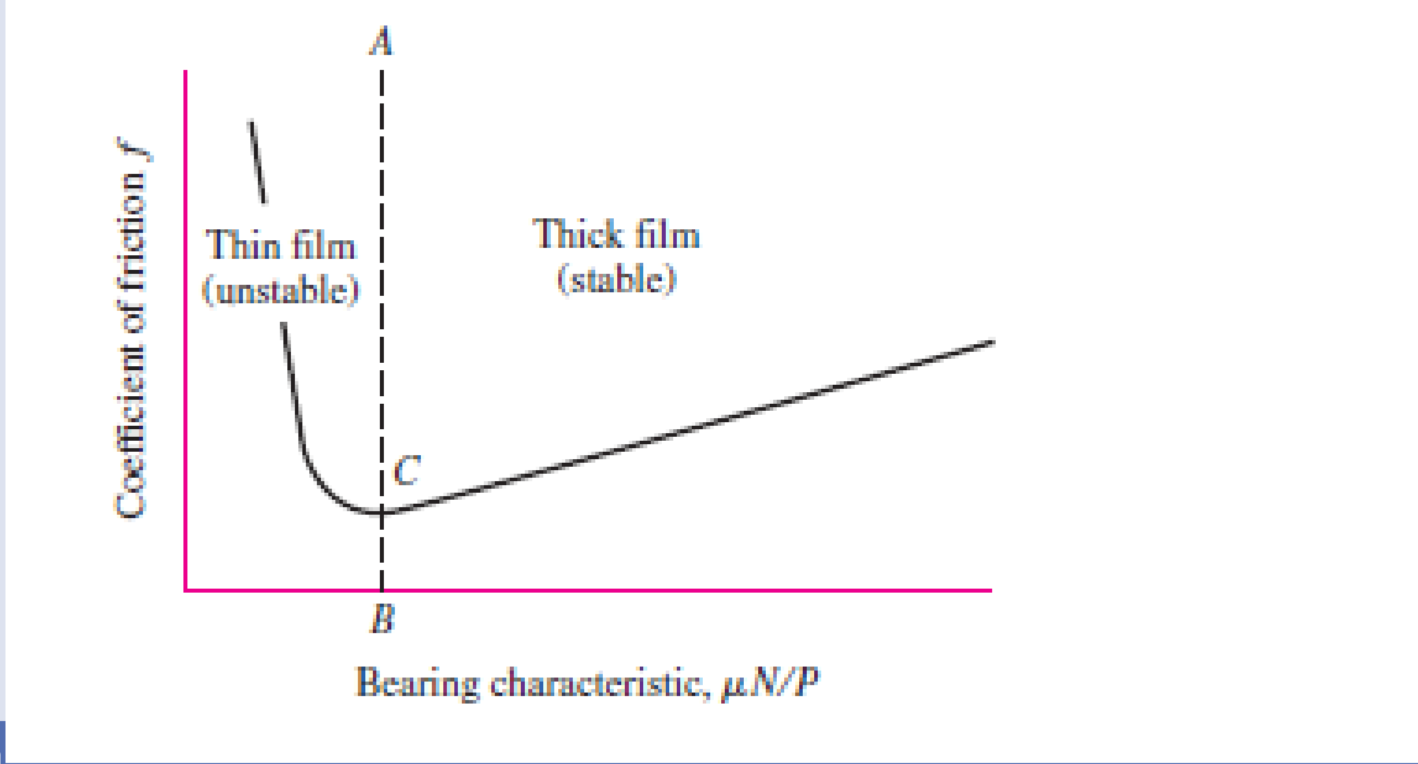
Stable Lubrication

▪



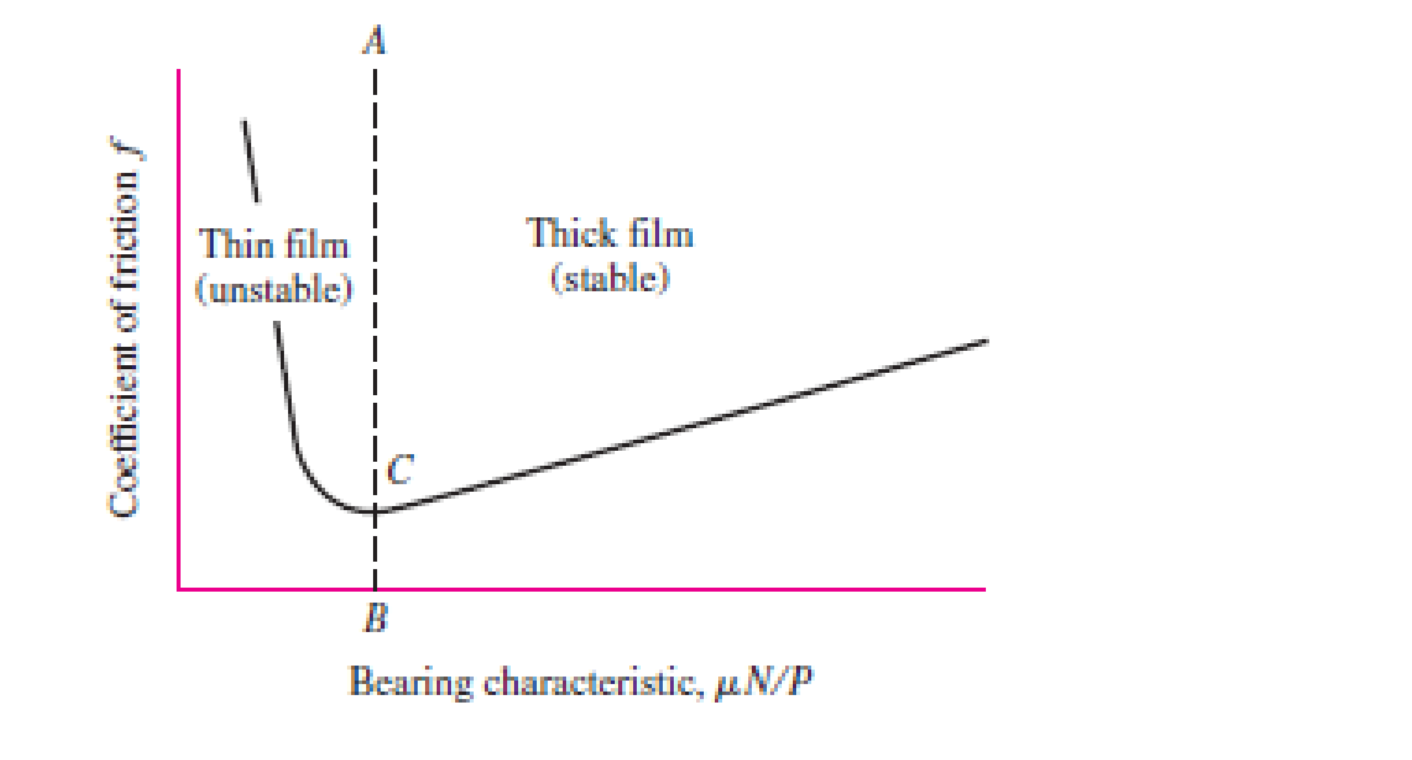
Stable Region (Right of line BA)

- Operating to the right of line *B A* and something happens, say, an increase in lubricant temperature.
  - This results in a lower viscosity and hence a smaller value of $\mu N/P$.



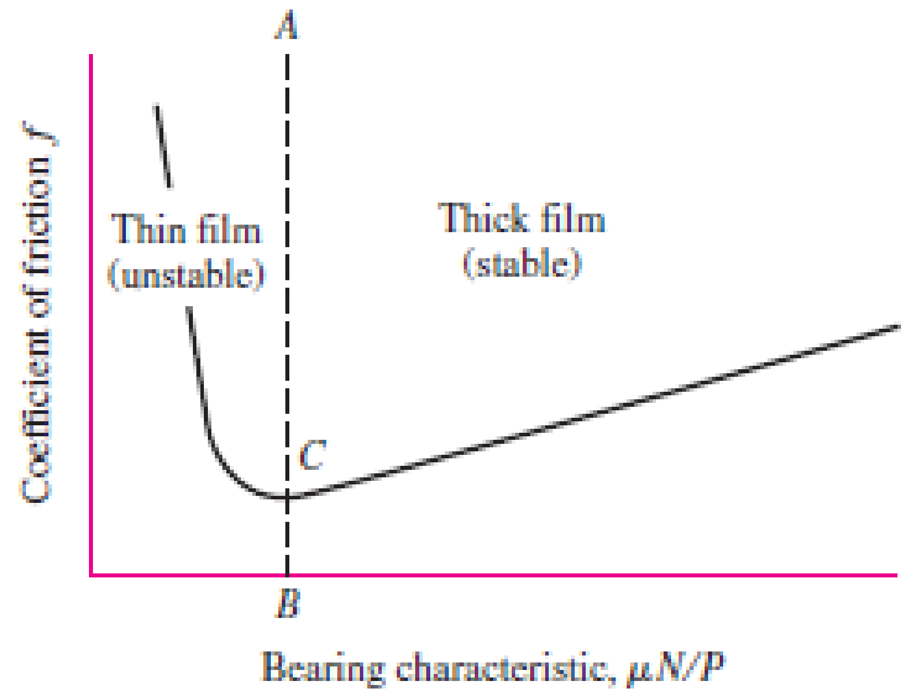
Stable Region (Right of line BA)

- The coefficient of friction decreases, not as much heat is generated in shearing the lubricant, and consequently the lubricant temperature drops.
- Thus the region to the right of line $B\ A$ defines *stable lubrication* because variations are self-correcting.



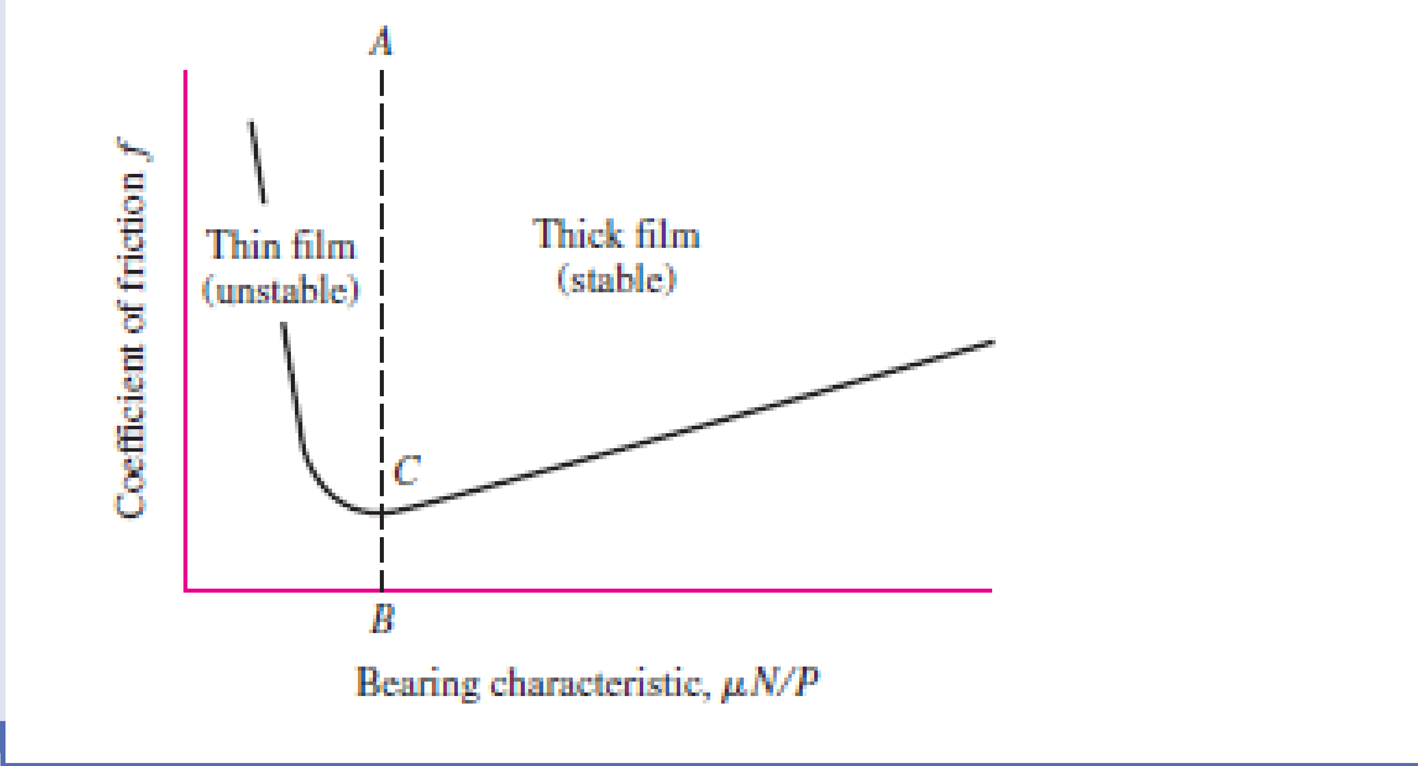
Unstable Region (Left of line BA)

- To the left of line *B A*,
  - A decrease in viscosity would increase the friction. A temperature rise would ensue, and the viscosity would be reduced still more. The result would be compounded.



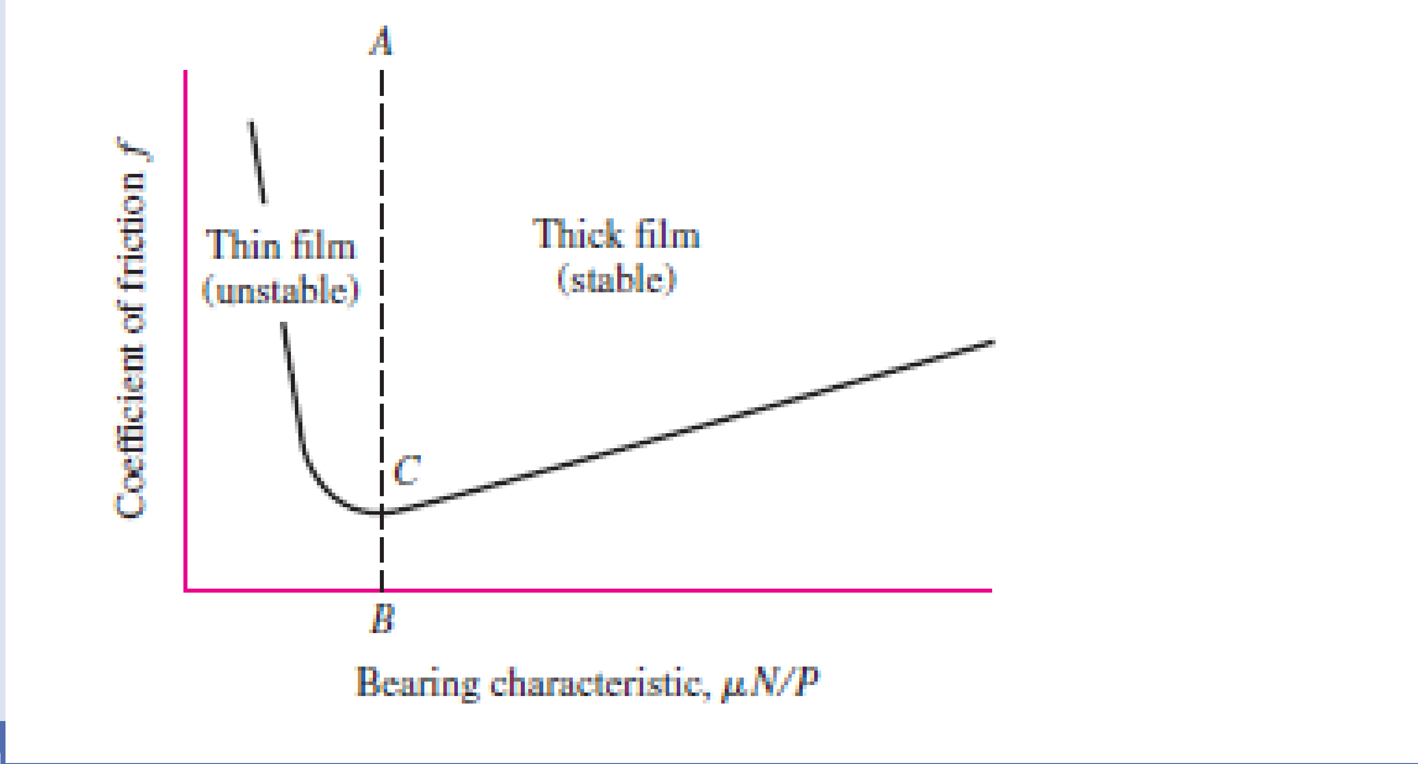
Unstable Region (Left of line BA)

- Thus the region to the left of line $B\ A$ represents *unstable lubrication*.
- It is also helpful to see that a small viscosity, and hence a small $\mu N/P$, means that the lubricant film is very thin and that there will be a greater possibility of some metal-to-metal contact, and hence of more friction.



On the line BA

- Thus, point *C* represents what is probably the beginning of metal-to-metal contact as μ*N*/*P* becomes smaller.



MACHINE DESIGN II
MEC 3110

Lecture-4

BY

PROF. M. NAUSHAD ALAM

MECHANICAL ENGINEERING DEPT.
A.M.U. ALIGARH

Hydrodynamic lubrication

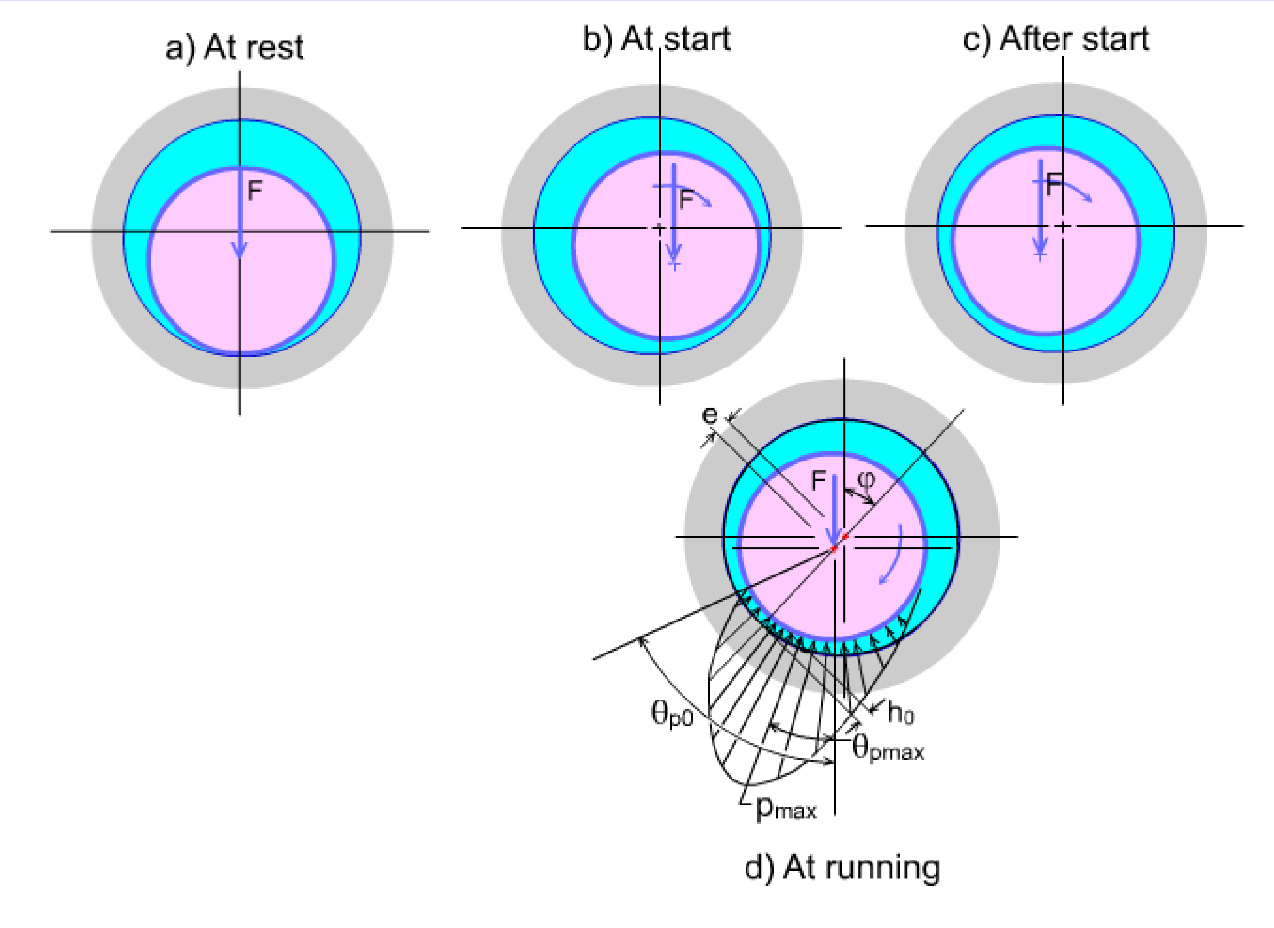


Fig.1.14 Journal position in hydrodynamic lubrication

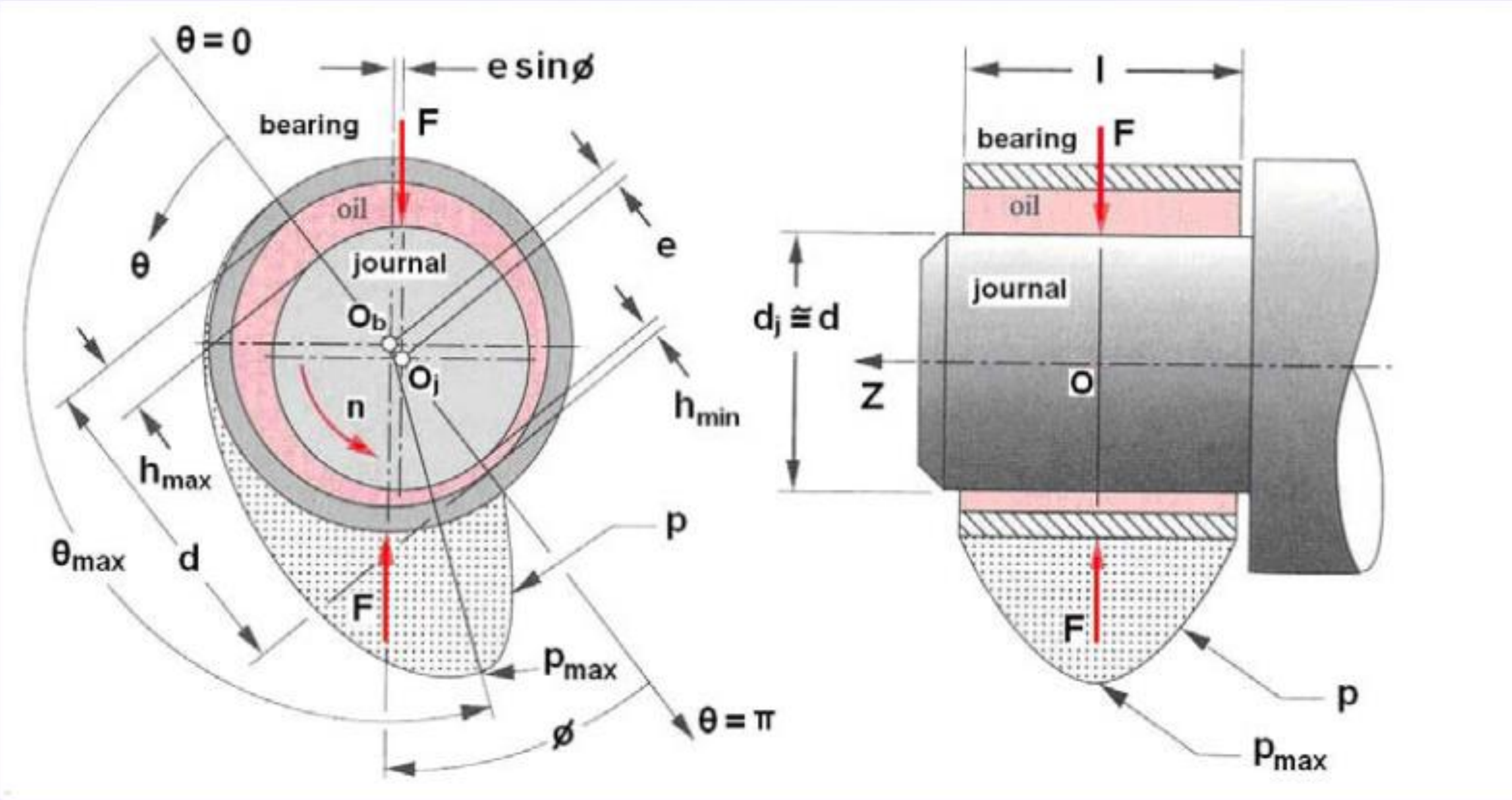


Fig.1.6 Hydrodynamic lubricated bearing

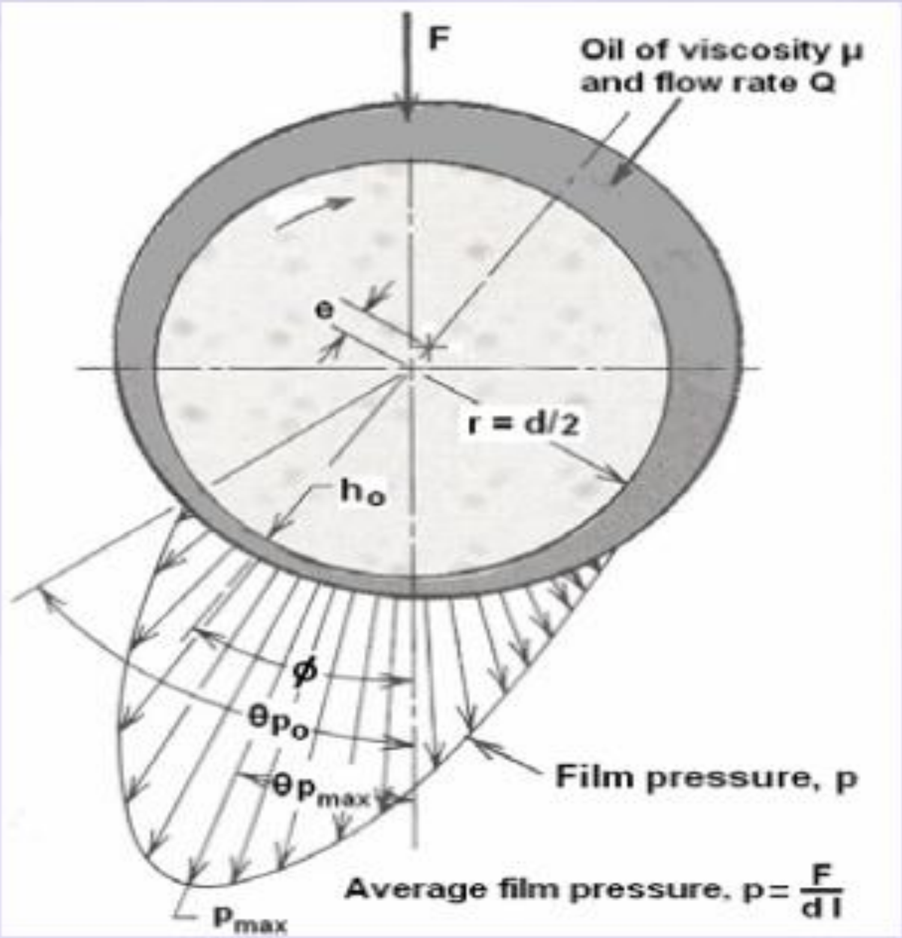


Fig.1.13 Stable hydrodynamic lubrication

1.6.5 LUBRICANT PROPERTIES

Properties of a good lubricant are:

1. It should give rise to low friction.
2. It should adhere to the surface and reduce the wear.
3. It should protect the system from corrosion.
4. It should have good cleaning effect on the surface.
5. It should carry away as much heat from the surface as possible.
6. It should have thermal and oxidative stability.
7. It should have good thermal durability.
8. It should have antifoaming ability.
9. It should be compatible with seal materials.
10. It should be cheap and available in plenty.

..

1.7.1 Recommended Lubricants for the Bearing Application

1. SAE 10 – spindle oil for light loaded bearings and high speeds.

2. SAE 20 – 40 – Machine oil for bearings of IC engines, machine tools, turbines etc.

3. SAE40-50 – Machine oil for diesel engines heavy load and medium speeds.

4. SAE 60-70 – machine oil for high temperature, heavy load and low speeds.

1.7.4 ISO Specification of Lubrication oils

Industrial fluid lubricants are commonly specified in terms of international standards, which appear as

1. ASTM D 2422,
2. American National Standard Z11.232,
3. ISO Standard 3448.

METHODS OF LUBRICANT SUPPLY

Lubricant present at the bearing surface gets depleted due to side leakage and to main the hydrodynamic lubrication continuous supply of lubricant must be ensured.

The principal methods of supply of lubricant are:

1. Oil Ring lubrication
2. Oil collar lubrication
3. Splash lubrication
4. Oil bath lubrication
5. Oil pump lubrication

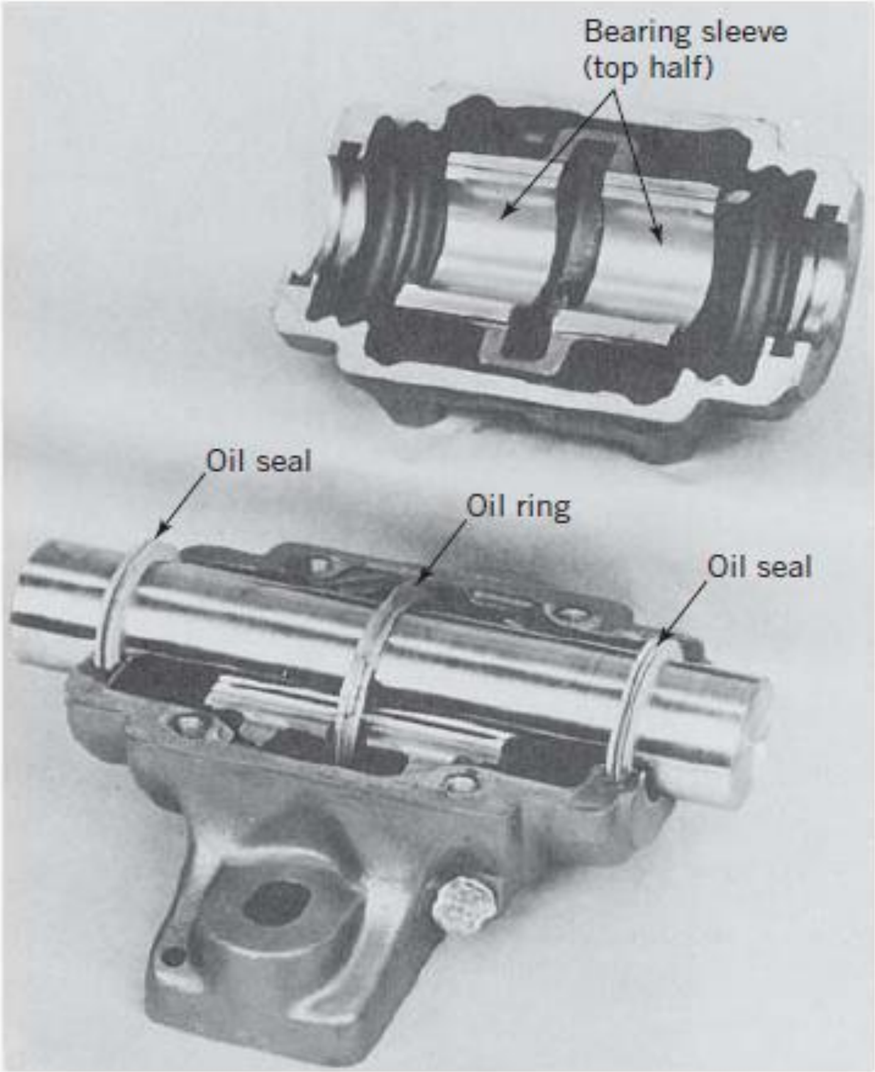
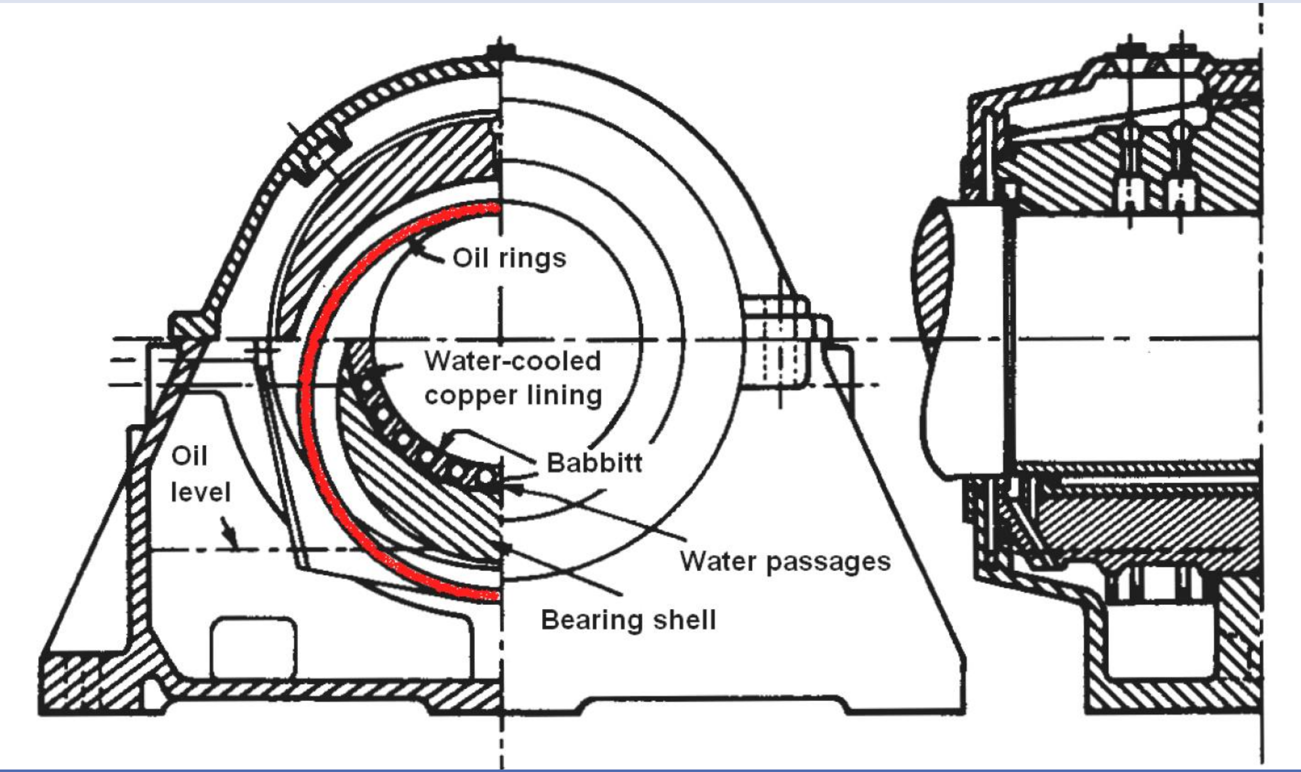


FIGURE 13.22
Ring-oiled bearing.
(Courtesy Reliance
Electric Company)

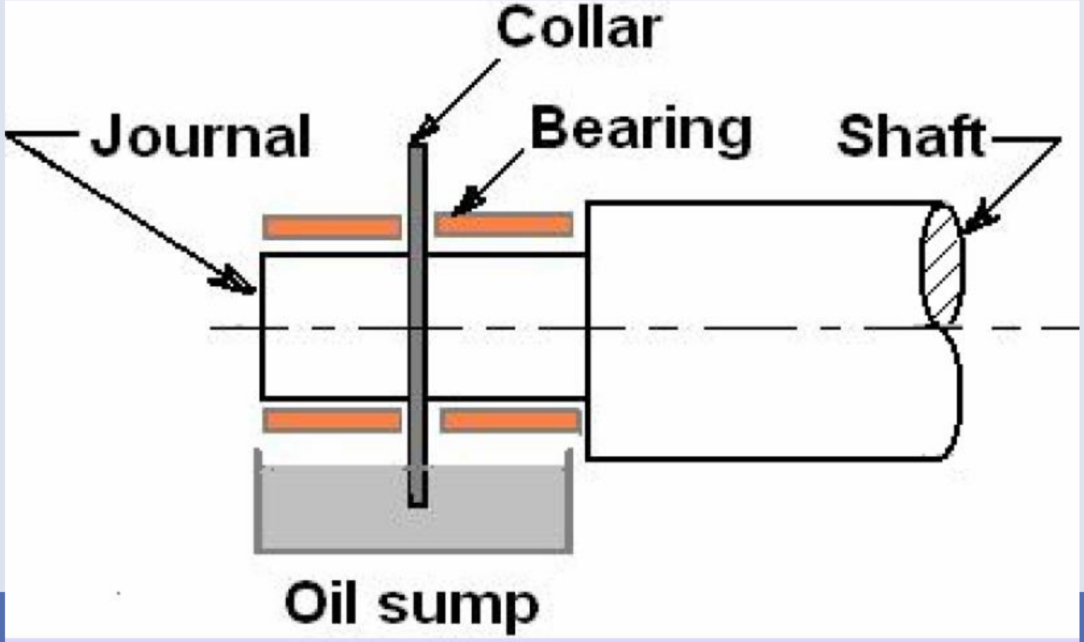
Oil Ring Lubrication

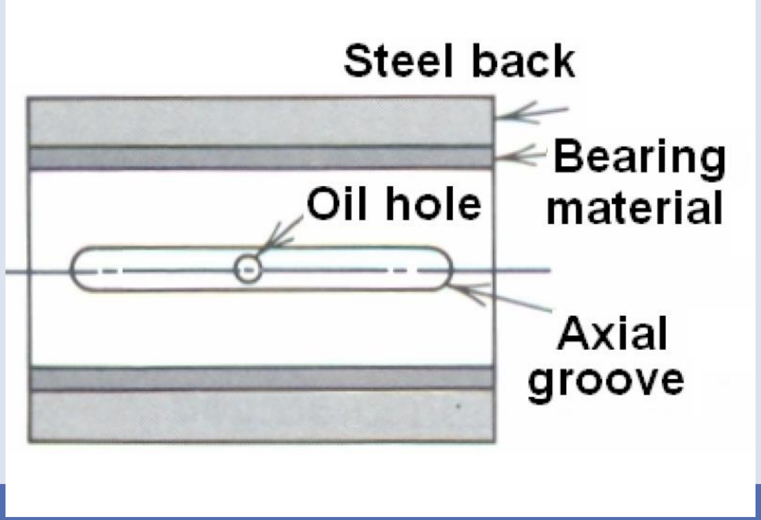
- The ring is of 1.5 to 2 times the diameter of the shaft.
- It hangs loosely on journal and rotates with the journal.
- As the ring rotates it lifts oil to the top.
- The bearing sleeve is slotted to accommodate the ring and bear against the journal.



Oil Collar Lubrication

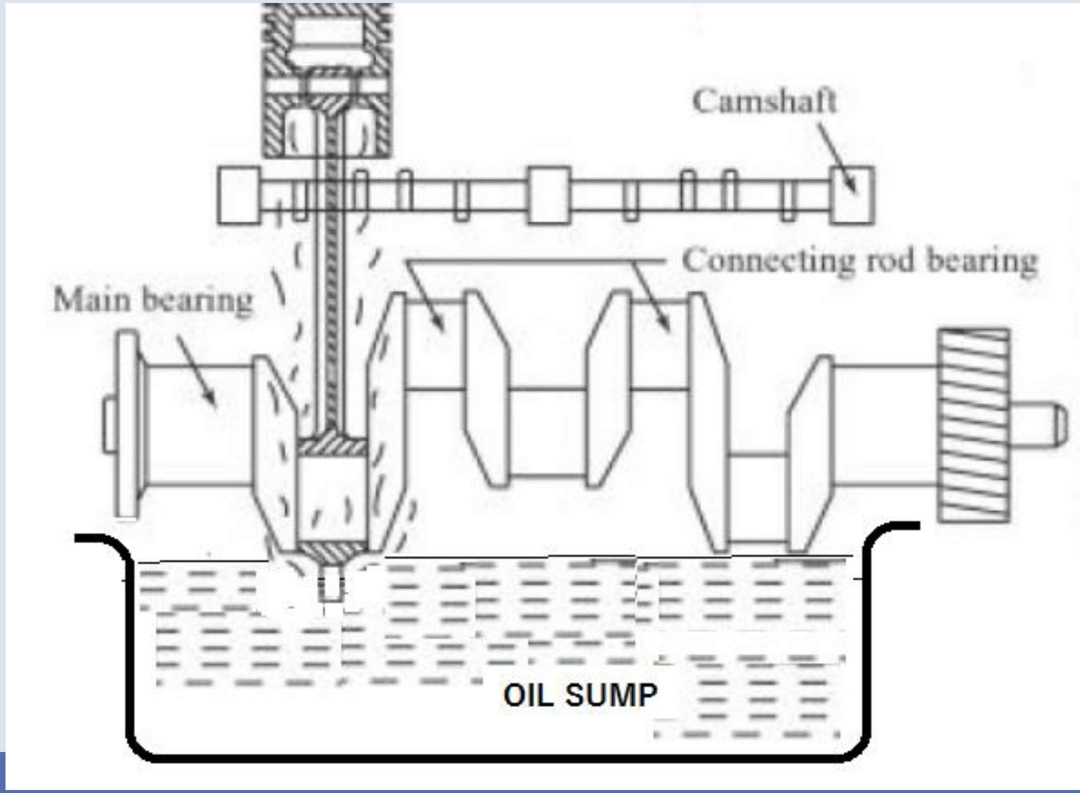
- A rigid collar integral with the journal dips into the reservoir at the bottom.
- A small sump is provided on either side of the collar.
- During rotation the collar carries oil to the top and throws off into the sumps.
- From the sump oil flows by gravity through the oil hole and groove to the bearing surfaces.





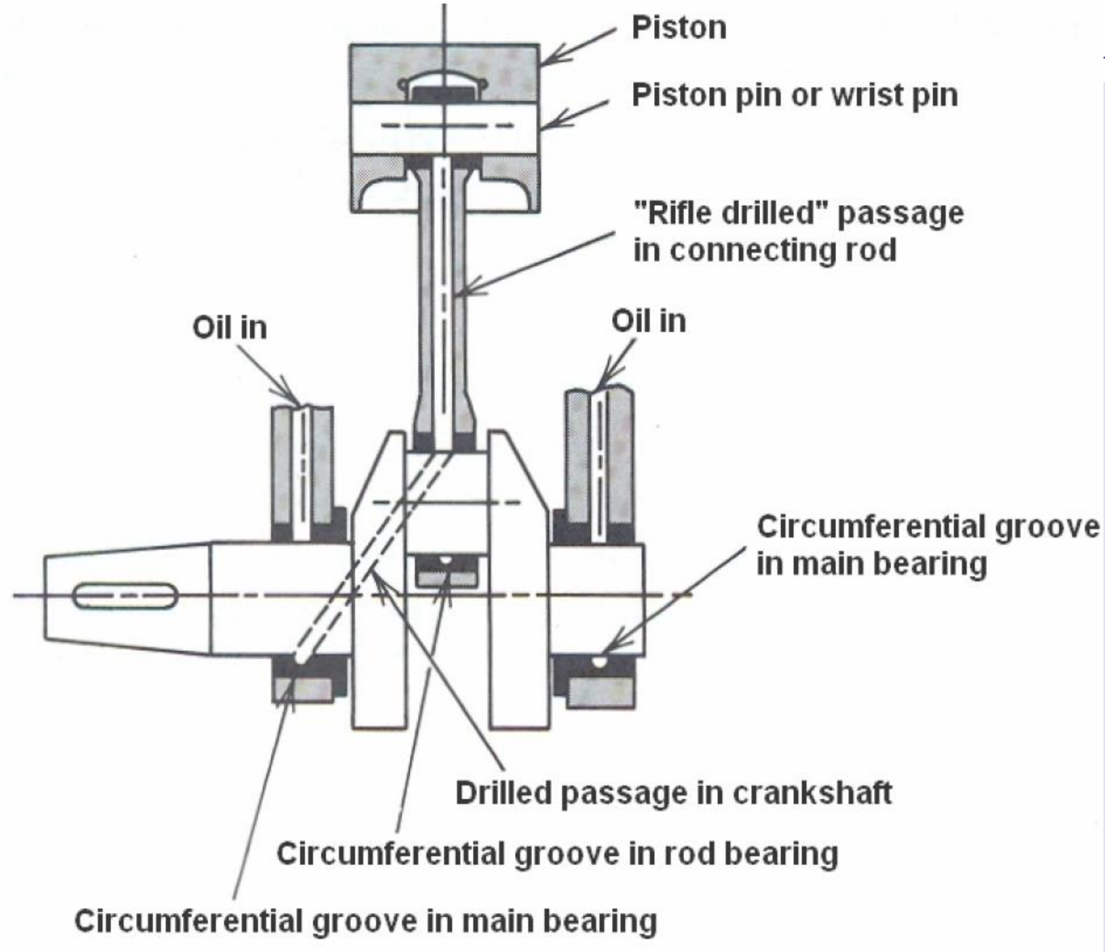
Splash Lubrication

- Oil is channelled to small sumps maintained above the bearings.
- The oil in sumps is splashed by moving parts or small oil scoops.
- The splashed oil is thrown onto the bearings for lubrication.
- Few examples are:
  - automobile engine
  - wrist pin lubrication
  - gearboxes lubrication
  - wherein gears splash
  - the oil into bearings.



Oil Pump Lubrication

- This is a positive means of supplying oil.
- Pumped oil fills the circumferential grooves in the main bearings.
- The holes in crankshaft carry oil to the connecting rod bearings.
- Circumferential groove
  transmits the oil through
  riffle drilled holes to the
  wrist pin bearings.



- **Pressure fed lubrication system of a piston engine or Compressor .**

End of Lecture

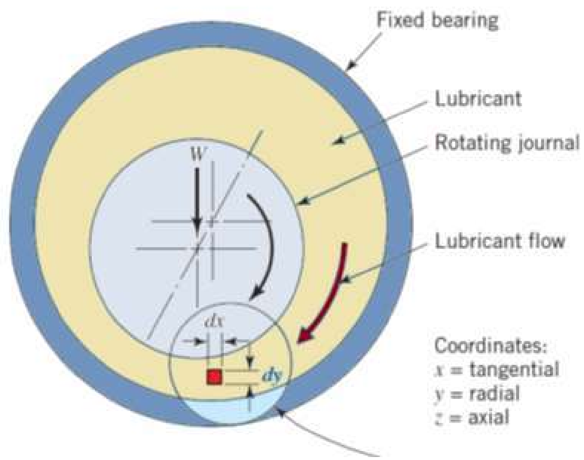
Any Questions

INTERACTION IS HIGHLY ENCOURAGED

Lecture - 5
Hydrodynamic Lubrication Theory

By
Prof. M. Naushad Alam

MACHINE DESIGN II
MEC 3110



MECHANICAL ENGINEERING DEPT.
A.M.U. ALIGARH

Hydrodynamic Lubrication Theory

- Theoretical analysis of hydrodynamic lubrication is traced to Osborne Reynolds's study of the laboratory investigation of railroad bearings by ***Beauchamp Tower*** in England during early 1880s.
- An oil hole was drilled to test the effect of adding an oiler at this point.
- Tower was surprised to discover that when the test device was operated without the oiler installed, oil flowed out of the hole!
- **Tower** tried to block the oil flow by pounding cork and wooden stoppers into the hole, but the hydrodynamic pressure forced them out.

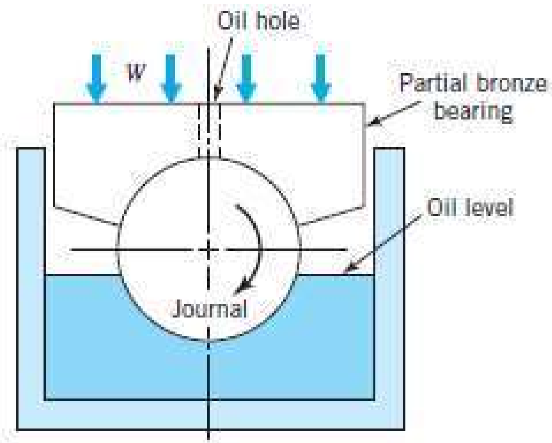


FIGURE 13.10
Schematic representation of Beauchamp Tower's experiment.

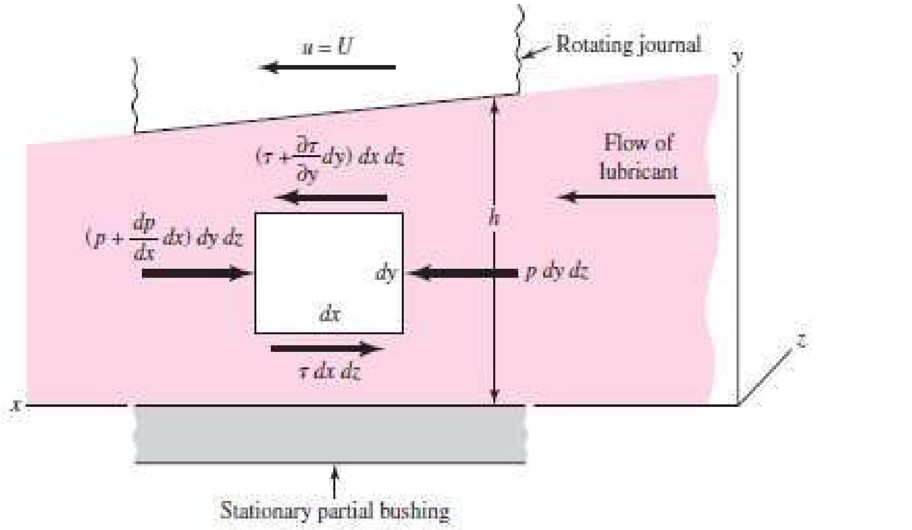
Hydrodynamic Lubrication Theory

- At this point, **Tower** connected a pressure gage to the oil hole and subsequently made experimental measurements of the oil film pressures at various locations.
- He then discovered that the summation of local hydrodynamic pressure times differential projected bearing area was equal to the load supported by the bearing.

- Reynolds's theoretical analysis led to his fundamental equation of hydrodynamic lubrication.

Derivation of the Reynolds equation

- Consider one dimensional flow between flat plates.
- One-dimensional flow means we are neglecting bearing side leakage.
- The analysis is approximately valid for bearings with L/D ratios greater than 1.5.
- This analysis can also be applied to journal bearings because the journal radius is so large in comparison to oil film thickness.



Reynolds Assumptions

- The fluid should meet following conditions; assumptions made:
  - Newtonian,
  - Incompressible,
  - Constant viscosity through-out the film
  - No pressure variation in the axial direction
  - No gravitational forces
  - Laminar flow,

e) The bearing and journal extend infinitely in the z direction. i.e., no lubricant flow in the z direction.

f) The film pressure is constant in the y direction. Thus the pressure depends on the x coordinate only.

g) The velocity of particle of lubricant in the film depends only on the coordinates x and y.

- the film is so thin that (1) it experiences negligible pressure variation over its thickness, (2) the journal radius can be considered infinite in comparison.

- The equation for equilibrium of forces in the x direction acting on the fluid element shown is:

$$p\,dy\,dz + \tau\,dx\,dz - \left(p + \frac{dp}{dx}dx\right)dy\,dz - \left(\tau + \frac{\partial \tau}{\partial y}dy\right)dx\,dz = 0 \quad \textbf{(a)}$$

The above equation reduces to

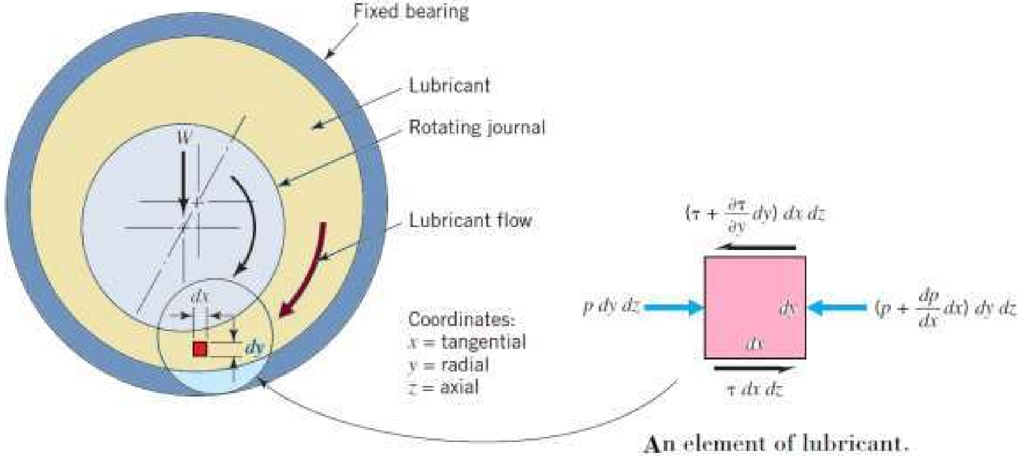
$$\frac{dp}{dx} = \frac{\partial \tau}{\partial y} \quad \textbf{(b)}$$

$$\tau = \mu\frac{\partial u}{\partial y} \quad \textbf{(c)}$$

$$\frac{dp}{dx} = \mu\frac{\partial^2 u}{\partial y^2}$$

or

$$\frac{\partial^2 u}{\partial y^2} = \frac{1}{\mu}\frac{dp}{dx}$$



An element of lubricant.

X components of Pressure and viscous forces

- Holding x constant and integrating twice with respect to y

$$\frac{\partial u}{\partial y} = \frac{1}{\mu}\left(\frac{dp}{dx}y + C_1\right)$$

$$u = \frac{1}{\mu}\left(\frac{dp}{dx}\frac{y^2}{2} + C_1 y + C_2\right) \qquad \textbf{(d)}$$

Assumption of no slip between the lubricant and the boundary surfaces gives boundary conditions to find *C*1 and *C*2

$$u = 0 \quad \text{at} \quad y = 0, \qquad u = U \quad \text{at} \quad y = h$$

Hence,

$$C_1 = \frac{U\mu}{h} - \frac{h}{2}\frac{dp}{dx}$$

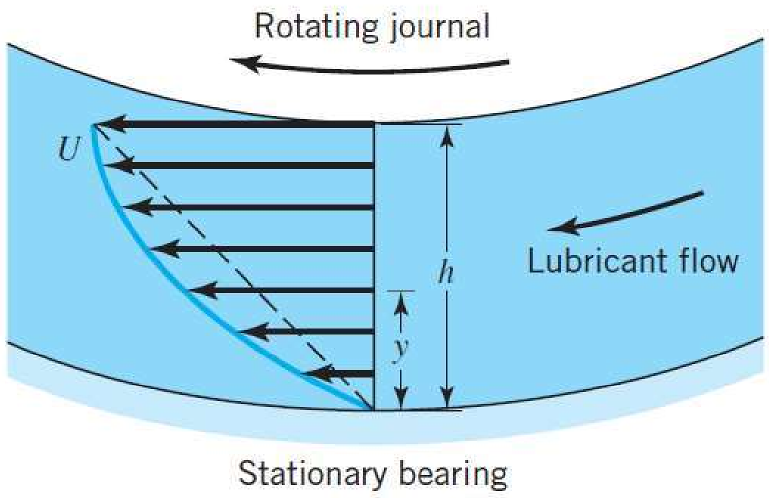
and $C_2 = 0$

- Substitution of the values *C*1 and *C*2, in Eq. ***d*** gives

$$u = \frac{1}{2\mu}\frac{dp}{dx}(y^2 - hy) + \frac{U}{h}y \qquad \textbf{(8)}$$

which is the equation for the velocity distribution of the lubricant film across any *yz*-plane as a function of distance *y*, pressure gradient *dp*/*dx*, and surface velocity *U.*

- This velocity distribution consists of two terms:
  (1) a linear distribution
  (2) a superimposed parabolic distribution given by the first term.



Lubricant velocity gradient.

- Let the volume of ... ng the element in Figure be *Qf*.

$$Q_f = \int_0^h u\,dy = \frac{Uh}{2} - \frac{h^3}{12\mu}\frac{dp}{dx} \quad \textbf{(e)}$$

- For unit width in th

- For a non-compressible lubricant, the flow rate must be the same for all cross sections,

$$\frac{dQ_f}{dx} = 0$$

$$= \frac{U}{2}\frac{dh}{dx} - \frac{d}{dx}\left(\frac{h^3}{12\mu}\frac{dp}{dx}\right) = 0$$

Classical *Reynolds equation for one-dimensional flow*.

$$\boxed{\frac{d}{dx}\left(\frac{h^3}{\mu}\frac{dp}{dx}\right) = 6U\frac{dh}{dx}}$$

Reynolds equation for two-dimensional flow

- When fluid flow in the z direction is included (i.e., axial flow and end leakage), a similar development gives the :

$$\frac{\partial}{\partial x}\left(\frac{h^3}{\mu}\frac{\partial p}{\partial x}\right) + \frac{\partial}{\partial z}\left(\frac{h^3}{\mu}\frac{\partial p}{\partial z}\right) = 6U\frac{\partial h}{\partial x}$$

$$S = \left(\frac{r}{c}\right)^2 \frac{\mu N}{P}$$

Ocvirk's short bearing approximation.

- There is no general analytical solution to the equation

$$\frac{\partial}{\partial x}\left(\frac{h^3}{\mu}\frac{\partial p}{\partial x}\right)+\frac{\partial}{\partial z}\left(\frac{h^3}{\mu}\frac{\partial p}{\partial z}\right)=6U\frac{\partial h}{\partial x}$$

- Only approximate solutions have been obtained by using electrical analogies, mathematical summations, relaxation methods, and numerical and graphical methods.
- One of the important solutions is due to **Sommerfeld,** expressed as

$$\frac{r}{c}f=\phi\left[\left(\frac{r}{c}\right)^2\frac{\mu N}{P}\right]$$

where φ indicates a functional relationship.

- Sommerfeld found the functions for halfbearings and full bearings by using the assumption of no side leakage.

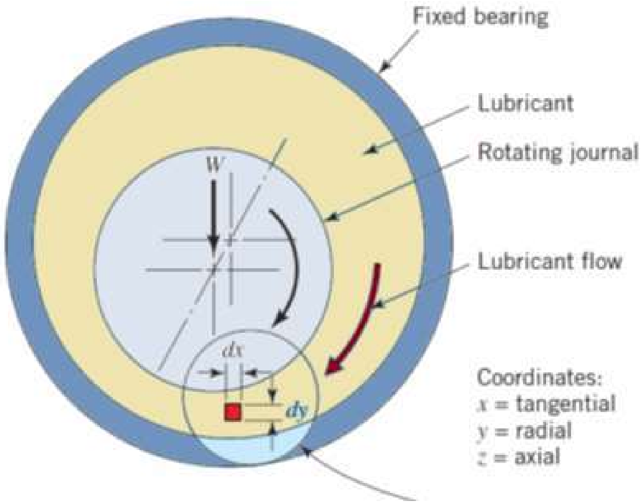
Lecture - 6

Hydrodynamic Lubrication

By

Prof. M. Naushad Alam

MACHINE DESIGN II
MEC 3110



MECHANICAL ENGINEERING DEPT.
A.M.U. ALIGARH

Design Considerations

- ❑ Two groups of variables are considered in the design of sliding bearings.

- In the **first group** are those whose values either given or are under the control of the designer. These are:

  1 The viscosity μ

  2 The load per unit of projected bearing area, P

  3 The speed N

  4 The bearing dimensions r, c, β, and l

  - ➢ Of these four variables, the designer usually has no control over the speed, because it is specified by the overall design of the machine.
  - ➢ Sometimes the viscosity is specified in advance.
  - ➢ The remaining variables, are therefore the decisions the designer makes.

…

- In the **second group** are the dependent variables.

The designer cannot control these except indirectly by changing one or more of the first group. These are:

1 The coefficient of friction *f*

2 The temperature rise *T*

3 The volume flow rate of oil *Q*

4 The minimum film thickness *h*0

Design Charts for Hydrodynamic Bearings

- Solutions of the *Reynolds equatio* $$\frac{d}{dx}\left(\frac{h^3}{\mu}\frac{dp}{dx}\right) = 6U\frac{dh}{dx}$$

  were first developed in the first decade of the twentieth century.

- Although theoretically applicable only to bearings that are "infinitely long", these solutions give reasonably good results with bearings of L/D ratios over about 1.5.

- The Ocvirk short bearing solution, based on Equation $$\frac{\partial}{\partial z}\left(\frac{h^3}{\mu}\frac{\partial p}{\partial z}\right) = 6U\frac{\partial h}{\partial x}$$

  is quite accurate for bearings of L/D ratios up to about 0.25, and is often used to provide reasonable approximations for bearings in the commonly encountered range of L/D between 0.25 and 0.75.

...

- Computerized solutions of the full Reynolds equation

$$\frac{\partial}{\partial x}\left(\frac{h^3}{\mu}\frac{\partial p}{\partial x}\right) + \frac{\partial}{\partial z}\left(\frac{h^3}{\mu}\frac{\partial p}{\partial z}\right) = 6U\frac{\partial h}{\partial x}$$

have been reduced to chart form by Raimondi and Boyd .

These provide accurate solutions for bearings of all proportions.

- Selected charts are reproduced in Figures
- Other Raimondi and Boyd charts apply to partial bearings (which extend around only 60°, 120°, or 180° of the journal circumference) and to thrust bearings.

..

- Values of any of the bearing performance variables plotted in coming Figures can be determined for any ratio of *L*/*D* greater than by using the following interpolation equation given by Raimondi and Boyd.

$$y = \frac{1}{(L/D)^3}\left[-\frac{1}{8}\left(1-\frac{L}{D}\right)\left(1-\frac{2L}{D}\right)\left(1-\frac{4L}{D}\right)y_\infty \right.$$
$$+\frac{1}{3}\left(1-\frac{2L}{D}\right)\left(1-\frac{4L}{D}\right)y_1-\frac{1}{4}\left(1-\frac{L}{D}\right)\left(1-\frac{4L}{D}\right)y_{1/2}$$
$$\left.+\frac{1}{24}\left(1-\frac{L}{D}\right)\left(1-\frac{2L}{D}\right)y_{1/4}\right]$$

- where *y* is the desired performance variable for any *L*/*D* ratio greater than 1/4 and *y*1, *y*1/2 , and *y*1/4 are the values of that variable for bearings having *L*/*D* ratios of q, 1, and respectively.

…

- Raimondi and Boyd charts apply to partial bearings (which extend around only 60°, 120°, or 180° of the journal circumference) and to thrust bearings.
- All the Raimondi and Boyd charts give plots of dimensionless bearing parameters as functions of the dimensionless bearing characteristic number, or Sommerfeld variable, S,

$$\text{Bearing characteristic number, } S = \left(\frac{R}{c}\right)^2 \frac{\mu n}{P}$$

The S scale on the charts is logarithmic except for a linear portion between 0 and 0.01.

Chart for minimum-film-thickness variable

$$\text{Bearing characteristic number, } S = \left(\frac{R}{c}\right)^2 \frac{\mu n}{P}$$

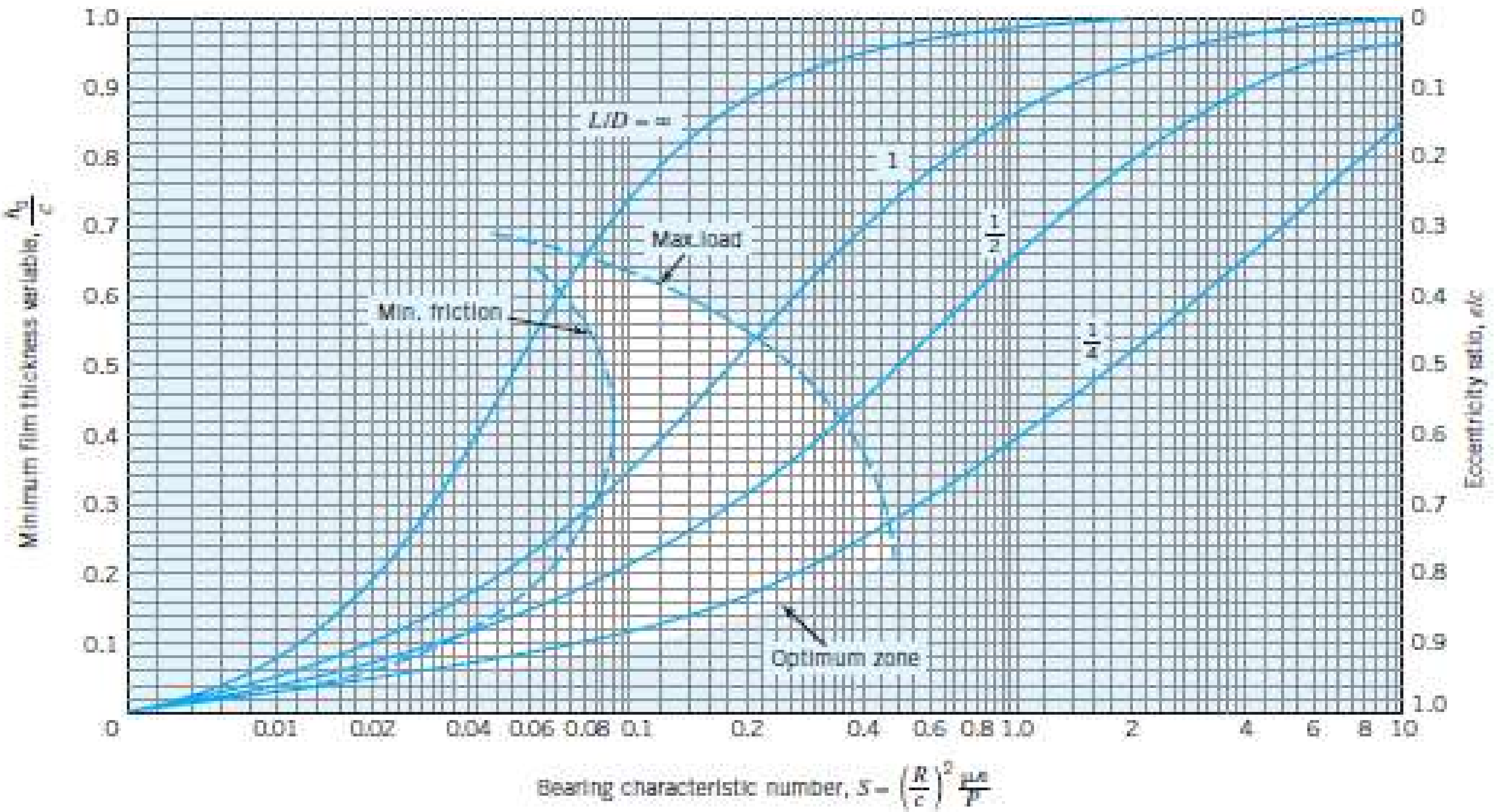


Chart for coefficient-of friction variable

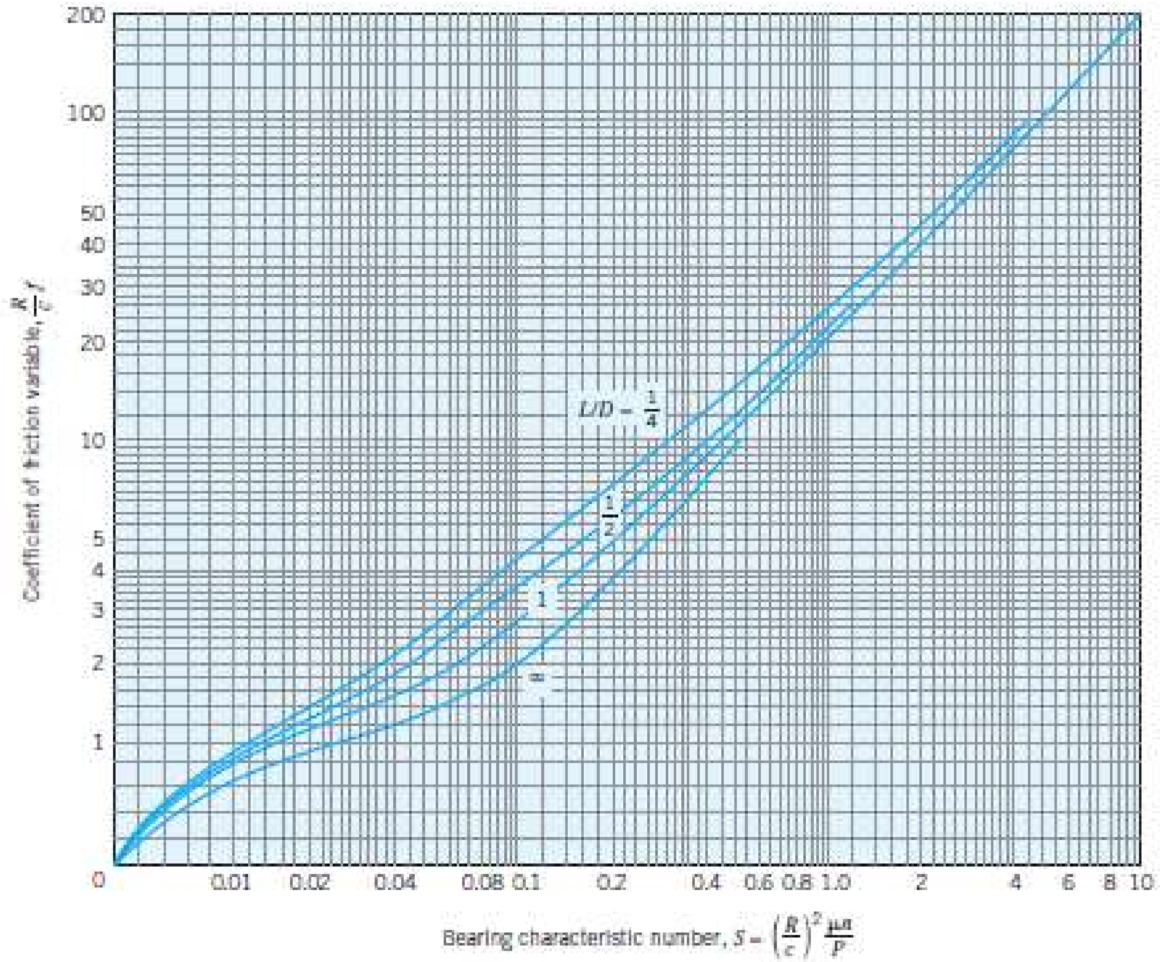


Chart for determining maximum film pressure

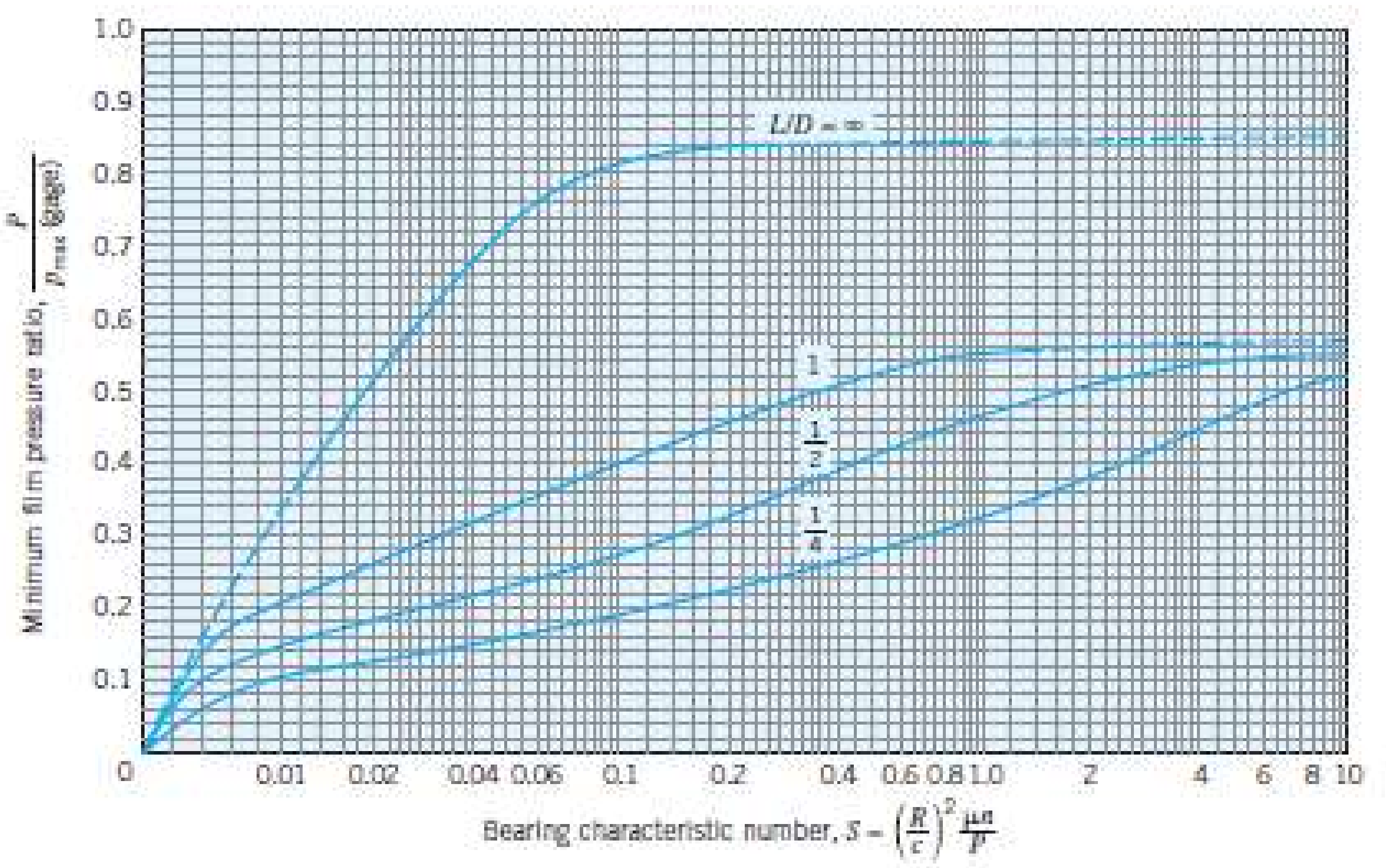
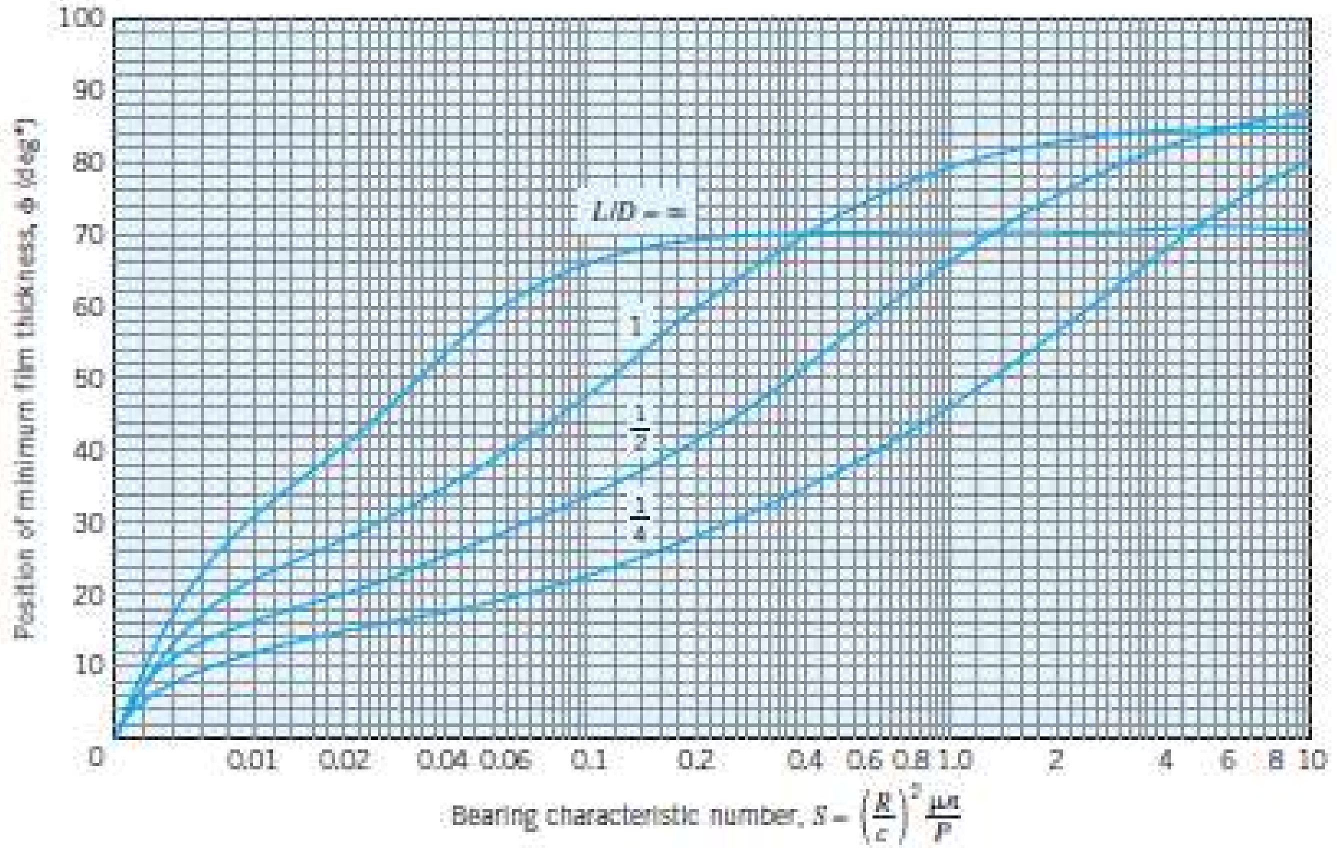


Chart for determining the position of the minimum film thickness *h*0



* Defined in Figure 13.20

Chart for positions of maximum film pressure and film termination

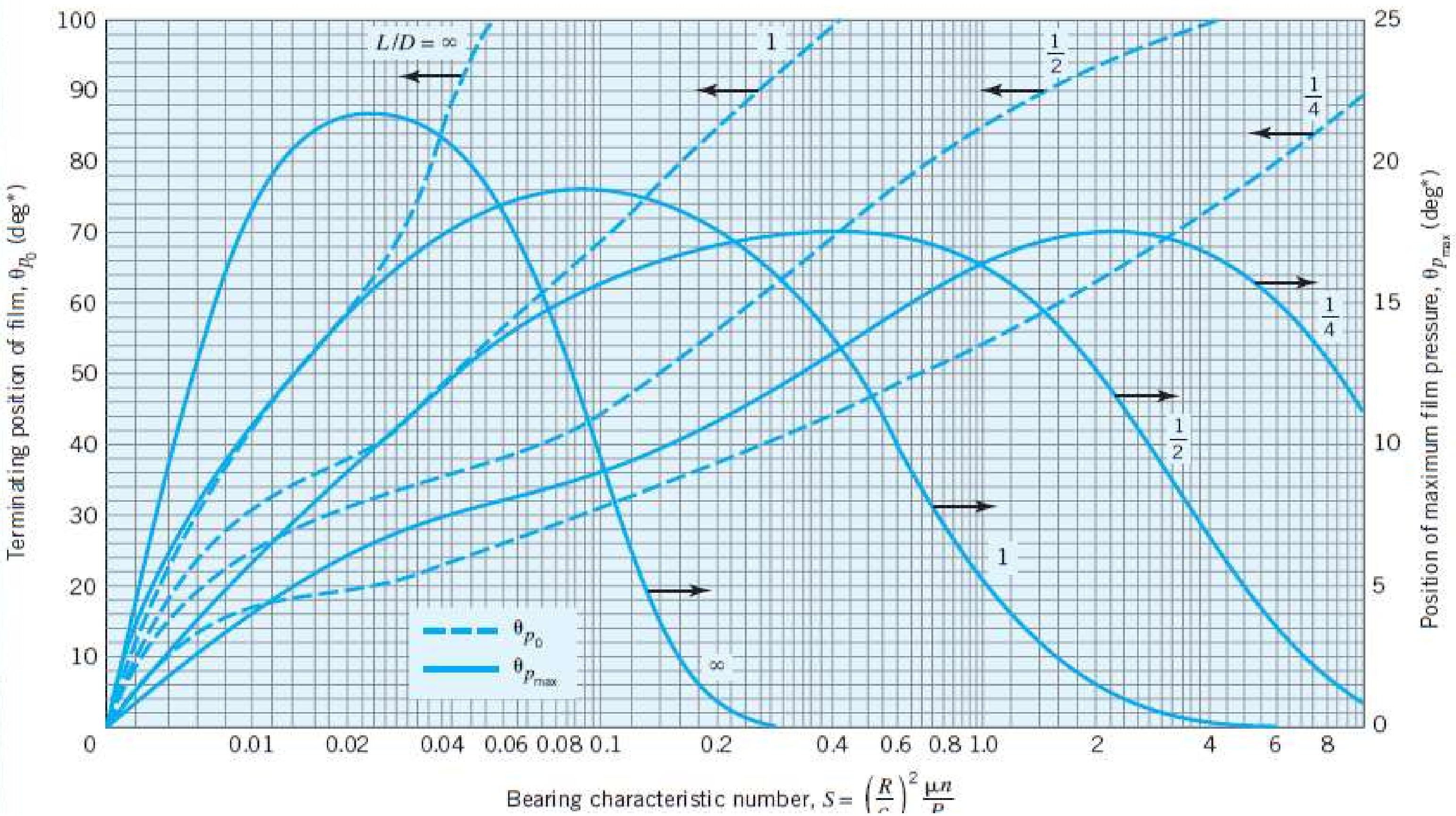


Chart for flow variable

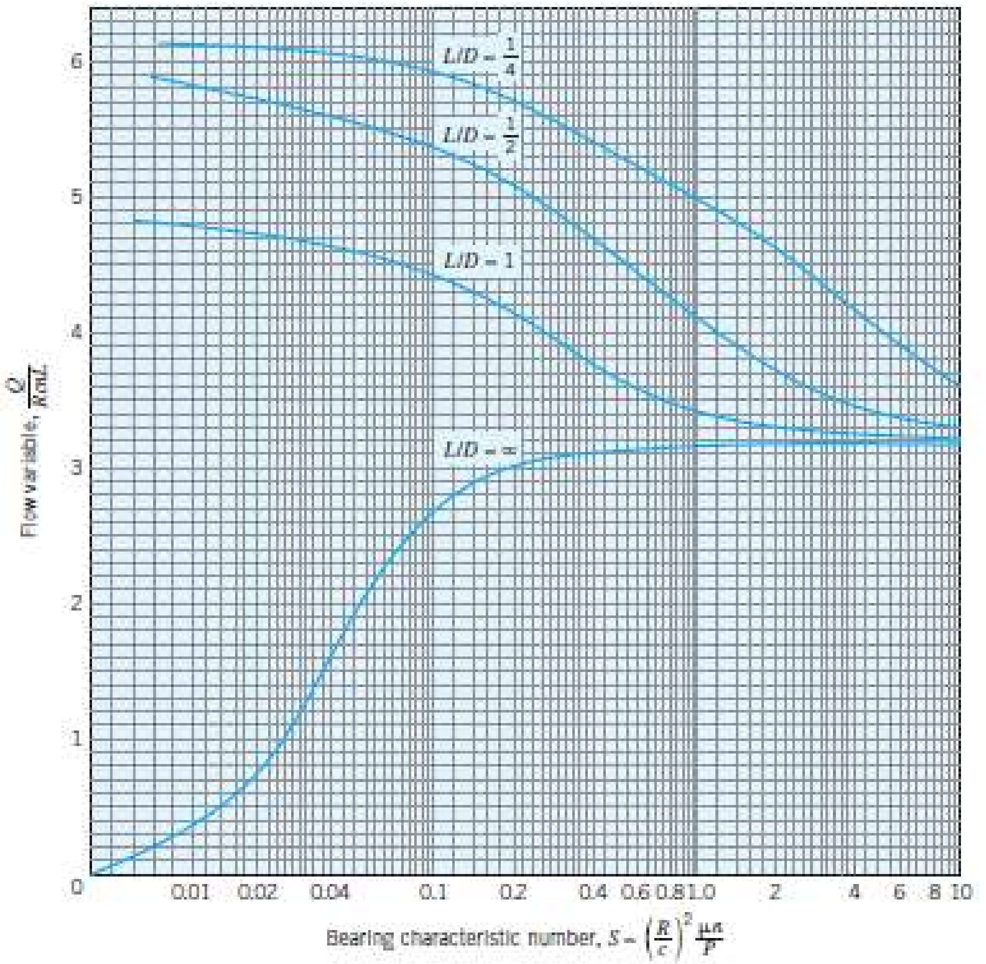
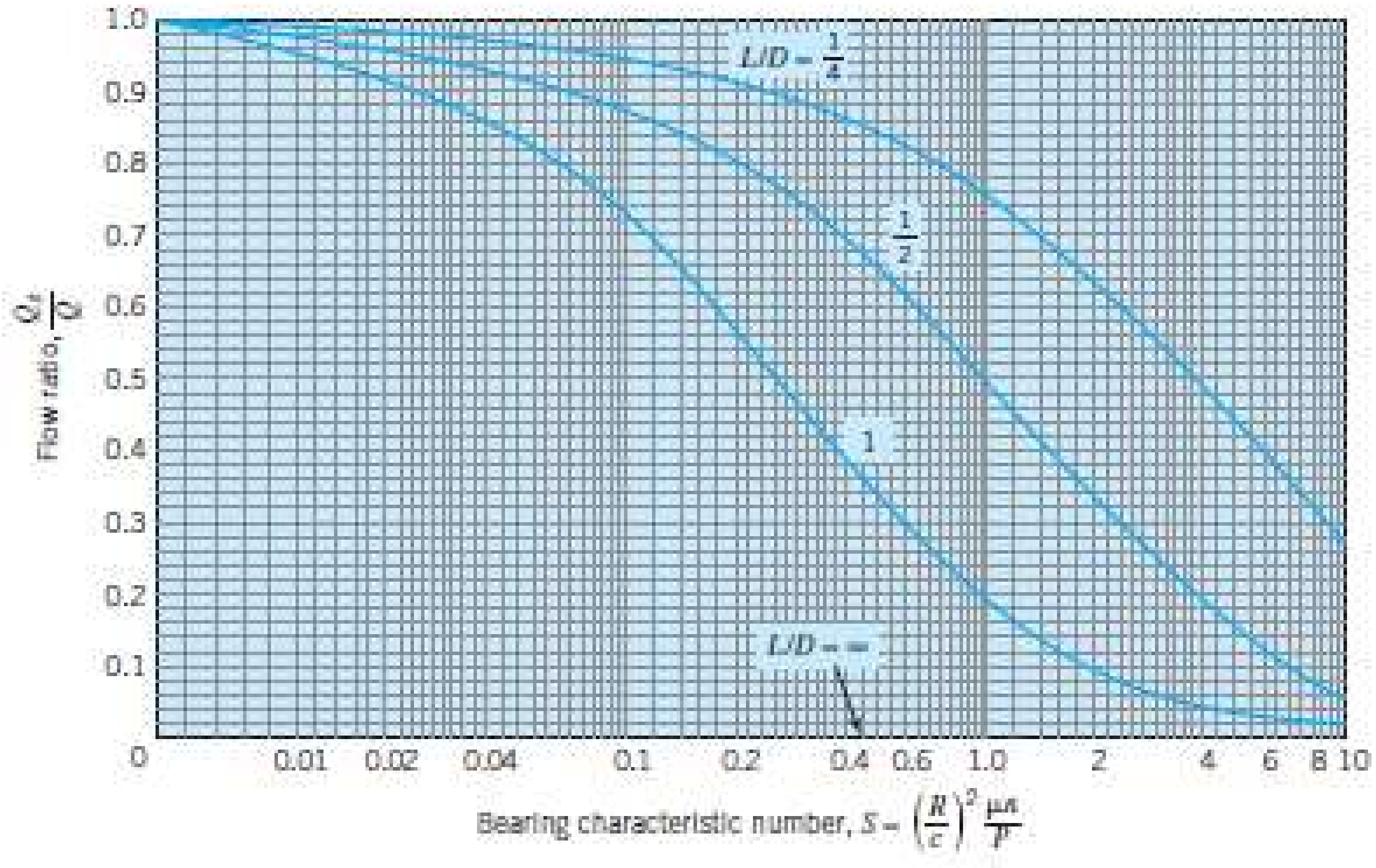
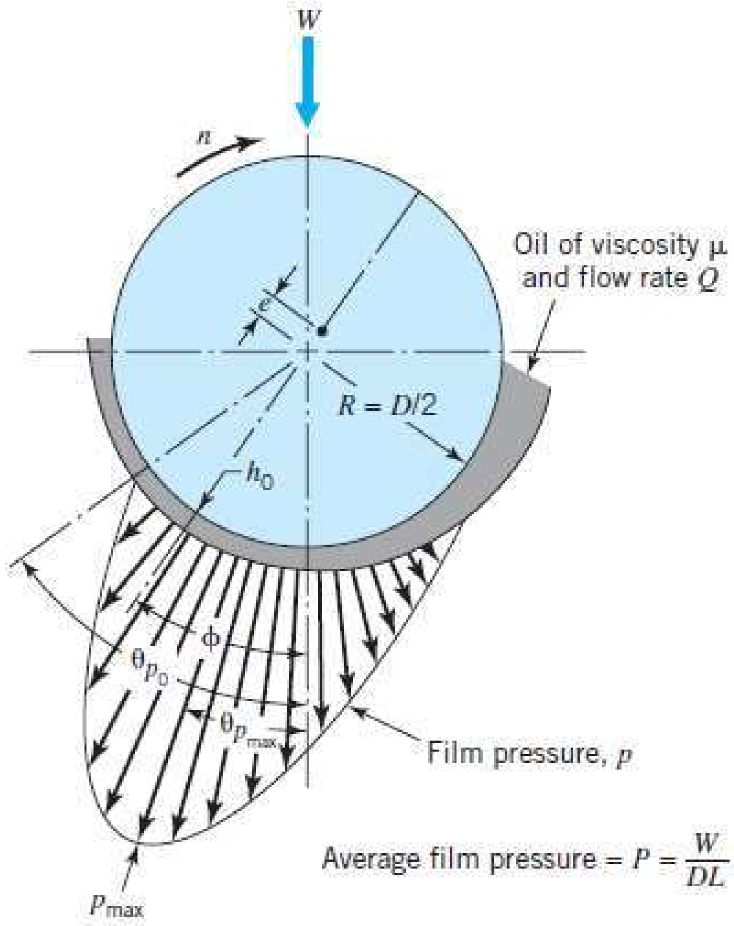


Chart for the ratio of side flow to total flow



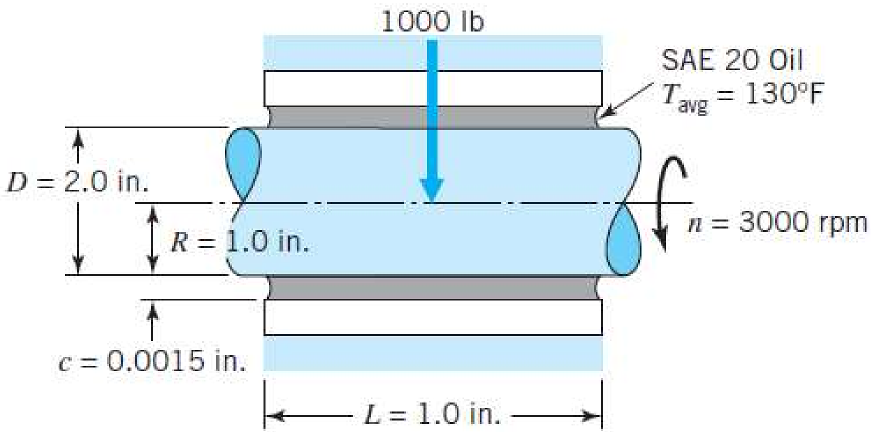
Polar diagram of film-pressure



PROBLEM - 1

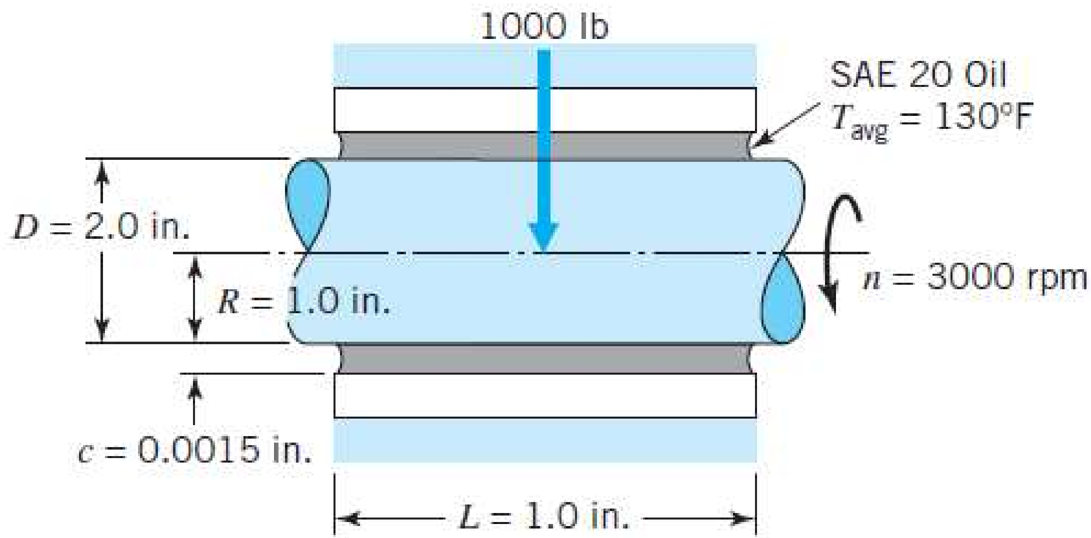
A journal bearing of 2-in. diameter, 1-in. length, and 0.0015-in. radial clearance supports a fixed load of 1000 lb when the shaft rotates 3000 rpm. It is lubricated by SAE 20 oil, supplied at atmospheric pressure. The average temperature of oil film is estimated at 130°F.

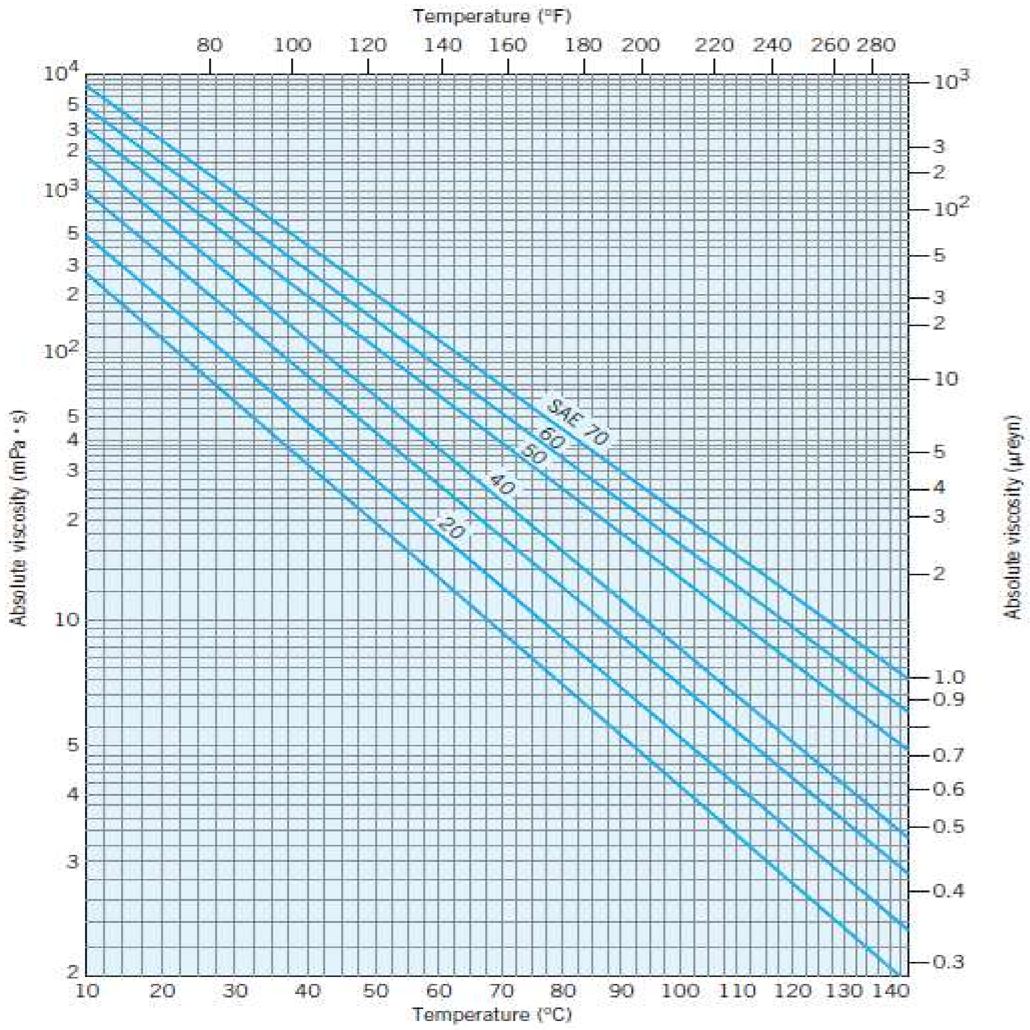
Using the Raimondi–Boyd charts, estimate the minimum oil film thickness, bearing coefficient of friction, maximum pressure within the oil film, angles ϕ, $\theta_{p_{\max}}$, and θ_{p_0}, and total oil flow rate through the bearing; the fraction of this flow rate that is recirculated oil flow; and the fraction of new flow that must be introduced to make up for side leakage.



Solution

- Assumptions:
- **1.** Bearing conditions are at steady state with the radial load fixed in magnitude and direction.
- **2.** The lubricant is supplied to the bearing at atmospheric pressure.
- **3.** The influence on flow rate of any oil holes or grooves is negligible.
- **4.** Viscosity is assumed to be constant and to correspond to the average of the oil flowing to and from the bearing.





Temperature (°F)
80
100
120
140
160
180
200
220
240
260
280
Absolute viscosity (mPa · s)
Absolute viscosity (μreyn)
SAE 70
60
50
40
20
10
20
30
40
50
60
70
80
90
100
110
120
130
140
Temperature (°C)

1. Given Data: $D = 2$ in.,

$R = 1$ in.,

$L = 1$ in.,

$c = 0.0015$ in.,

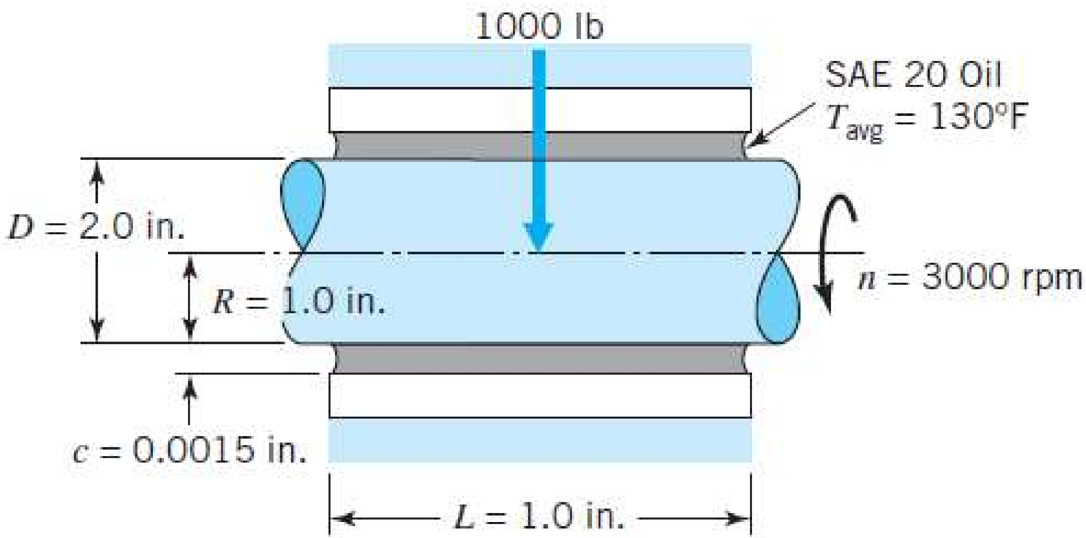
$n = 50$ rps,

$W = 1000$ lb.

2.

$$P = \frac{W}{LD} = \frac{1000}{(1)(2)} = 500 \text{ psi}$$

$$\mu = 4 \times 10^{-6} \text{ reyn} \quad \text{(Figure)}$$



…

3. Calculate S:

$$S = \left(\frac{R}{c}\right)^2\left(\frac{\mu n}{P}\right) = \left(\frac{1}{0.0015}\right)^2\frac{(4\times 10^{-6})(50)}{500} = 0.18$$

4. Use $S = 0.18$, $L/D = 0.5$ to enter all charts, and use units of inch-pound-seconds consistently:

From Figure $h_0/c = 0.3$, hence $h_0 = 0.00045$ in.

...

4. From the charts Calculate:

$$(R/c)f = 5.4, \text{ hence } f = 0.008.$$
$$P/p_{\max} = 0.32, \text{ hence } p_{\max} = 1562 \text{ psi}.$$
$$\phi = 40°.$$
$$\theta_{p_0} = 54°, \theta_{p_{\max}} = 16.9°.$$
$$Q/RcnL = 5.15, \text{ hence } Q = 0.39 \text{ in.}^3/\text{s}.$$
$$Q_s/Q = 0.81,$$

Hence side leakage that must be made up by “new” oil represents 81 percent of the flow; the remaining 19 percent is recirculated.

PROBLEM - 2

A machine journal bearing has a journal diameter of 150 mm and length of 120 mm. The bearing diameter is 150.24 mm. It is operating with SAE 40 oil at 65 C. The shaft is carrying a load of 8 kN and rotates at 960 rpm. Estimate the bearing coefficient and power loss using Petroff's equation.

Data Given :

$d = 0.15m;$

$D = 0.15024m;$

$l = 0.12\ m;$

$F = 8kN;$

SAE 40 oil $T_o = 65^oC;$

$n = 960/60 = 16$ rps.

To Find

$f = ?$, $N_{loss} = ?$

Solution

r = 0.5d = 0.5 x 0.15 = 0.075 m

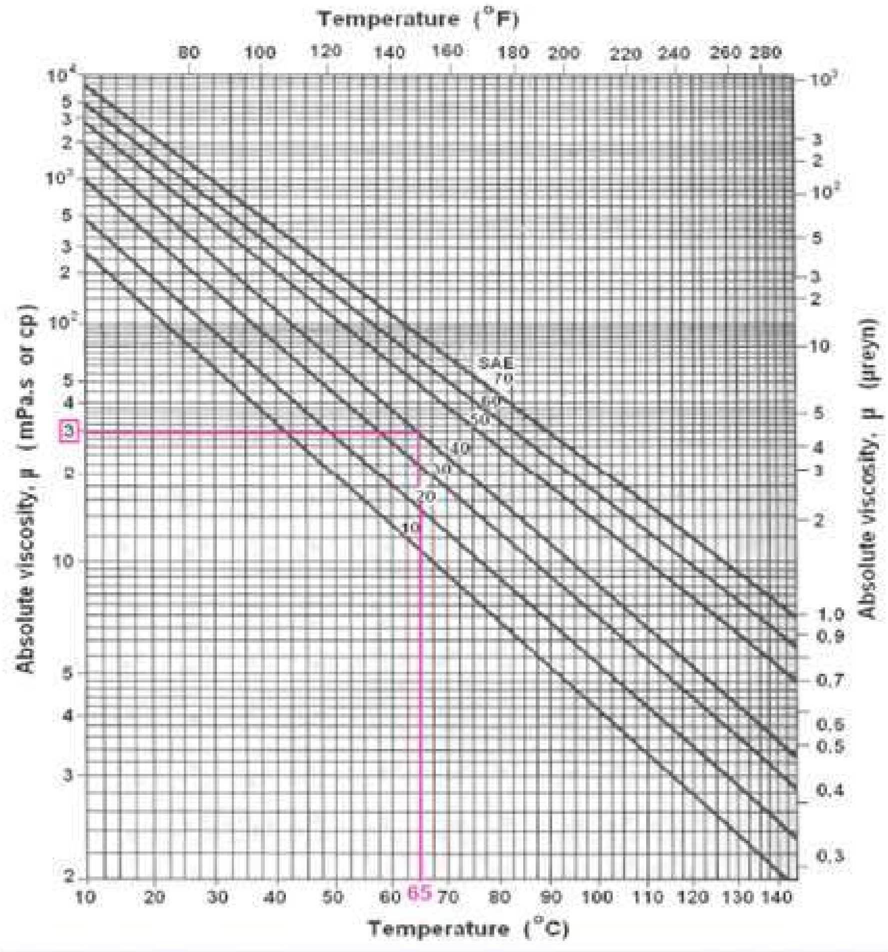
c = (D-d) /2 = 0.00012 m

p = F/dl = 8000/ 150x 120 = 0.44 MPa= 44 x 104 Pa

Viscosity of SAE 40 at 65°C, μ = 30 mPa.s = 30×10^{-3} Ns/m^2

(a) $$f = 2\pi^2\left(\frac{\mu n}{p}\right)\left(\frac{r}{c}\right)$$

$$= 2\pi^2\left(\frac{30\times10^{-3}\times16}{44\times10^{4}}\right)\left(\frac{0.075}{0.00012}\right) = 0.0134$$



Solution

(b) Friction Torque

$T_f = f\,F\,r$

$= 0.0134 \times 8000 \times 0.075$

$= 8.067$ Nm

$\omega = 2\pi n/60$

$= 2 \times 3.14 \times 960 / 60$

$= 100.48$ rad/s

Power loss:

$N_{loss} = T_f\,\omega$

$= 8.067 \times 100.48$

$= 811$ W

Lecture - 7

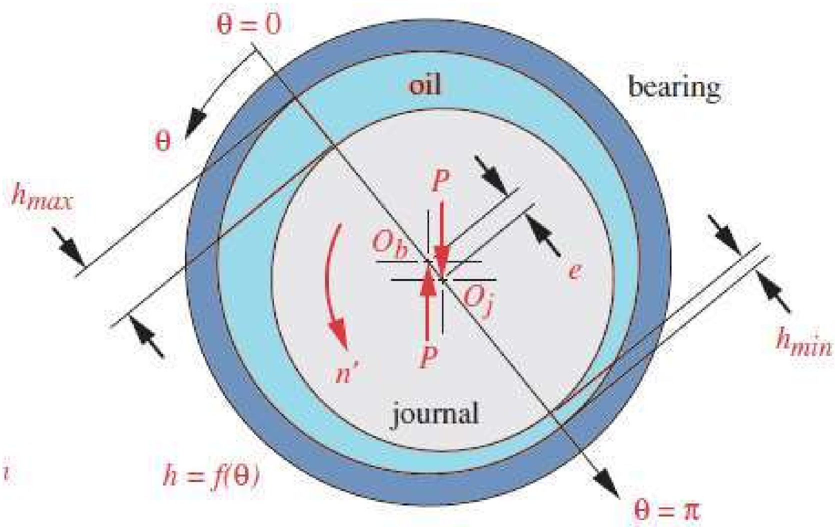
Hydrodynamic Lubrication

By

Prof. M. Naushad Alam

1

MACHINE DESIGN II
MEC 3110



MECHANICAL ENGINEERING DEPT.
A.M.U. ALIGARH

DESIGN CHARTS FOR HYDRODYNAMIC BEARINGS

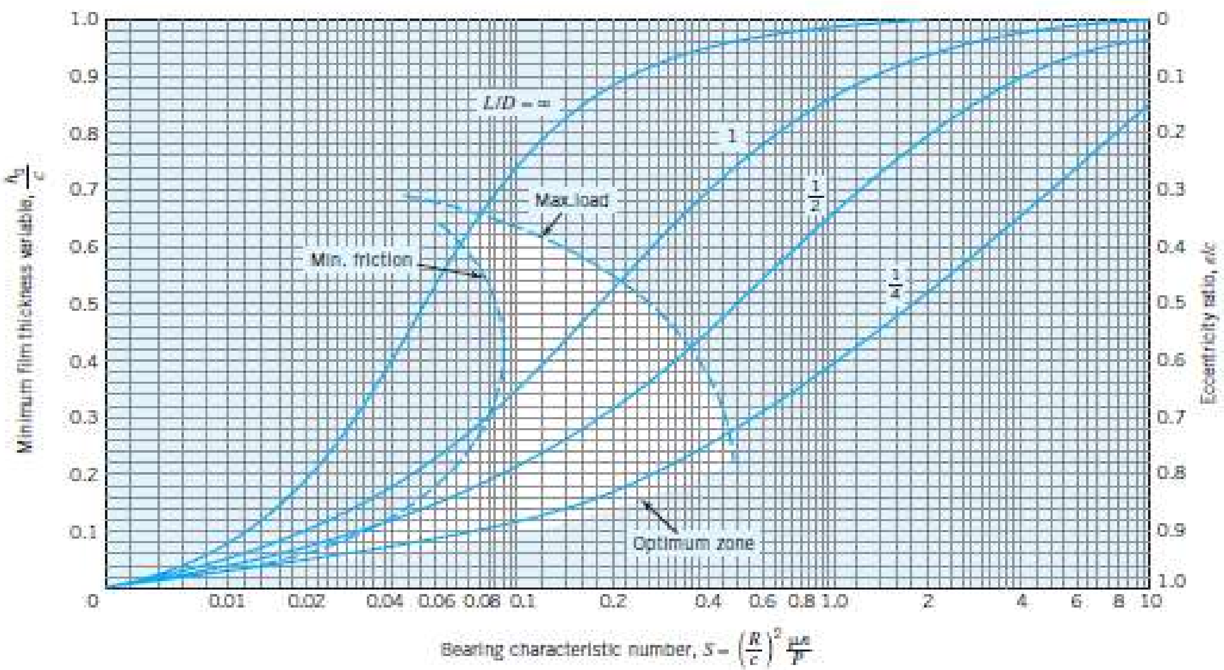
- Raimondi and Boyd charts are **plots of** ***dimensionless bearing parameters*** as functions of *the* ***dimensionless bearing characteristic number***, or Sommerfeld variable, S.
- Raimondi and Boyd charts apply to partial bearings (which extend around only 60°, 120°, or 180° of the journal circumference) and to thrust bearings.

$$\text{Bearing characteristic number, } S = \left(\frac{R}{c}\right)^2 \frac{\mu n}{P}$$

The S scale on the charts is logarithmic except for a linear portion between 0 and 0.01.

Raimondi and Boyd charts

- **Raimondi** and **Boyd** have obtained computerized solutions for ***Reynolds equation***, and reduced them to chart form.
- These charts provide accurate solutions for bearings of all proportions.
- Selected charts are shown in the Figures.



Problem

- A journal of a stationary oil engine is 80 mm in diameter. and 40 mm long. The radial clearance is 0.060mm. It supports a load of 9 kN when the shaft is rotating at 3600 rpm. The bearing is lubricated with SAE 40oil supplied at atmospheric pressure and average operating temperature is about 65oC. Using Raimondi- Boyd charts analyze the bearing **Data:** d = 80 mm; l =40 mm; c = 0.06 mm; F = 9kN;

- **Data Given:**

  n = 3600rpm = 60 rps;

  SAE 40 oil;

  $T_o = 65^oC$;

  Assuming that it is working under steady state condition.

Solution

Analysis:

1. p= F / ld

   = 9 x1000 /40 x 80 = 2.813 MPa

2. μ = 30 cP at 65oC for SAE 40 oil from Fig. 2.3a.

3. $$S=\left(\frac{r}{c}\right)^2\left(\frac{\mu n}{p}\right)$$

   $$=\left(\frac{40}{0.06}\right)^2\left(\frac{30\text{x}10^{-3}\text{ x }60}{2.813\text{ x }10^6}\right)=0.284$$

...

4. For $S = 0.284$ and $l/d = ½$,

$h_o/c = 0.38$

For $\varepsilon = e/c = 0.62$ from Fig.6.

$h_o = 0.38xc$

$= 0.382x\ 0.06$

$=0.023mm = 23\mu m$

$e = 0.62 \times c$

$= 0.62 \times 0.06 = 0.037$ mm

----

5. (r /c) f = 7.5,
   for S = 0.284 and l /d = ½ from Fig.2.11a.
   f = 7.5 x (c / r) = 7.5x (0.06/40) = 0.0113

6. Φ = 46o, for S = 0.284 for l /d = ½ from Fig.2.9a.

7. (Q / r c n l) = 4.9, for S = 0.284 for l /d = ½ from Fig.2.12a.
   Q = 4.9 r c n l = 4.9 x 0.04 x 0.00006 x 60 x 0.04
   = 2.82x10-5 m3/s
   = 28.2 cm3 /s

----

8. (Qs /Q) = 0.75, for S = 0.284 for l /d = ½ from Fig.2.13a.

   Qs = 0.75 Q = 0.75 x 28.2 = 21.2 cm3 /s

9. (p / p max) = 0.36, for S = 0.284 for l /d = ½ from Fig.2.14a.
   puma = p /0.36 = 2.813 / 0.36 = 7.8 MPa

10. θpox = 61.5o and θpuma = 17.5o, for S = 0.284 for l /d = ½ from Fig.2.15a.

. Temp-Viscosity Chart

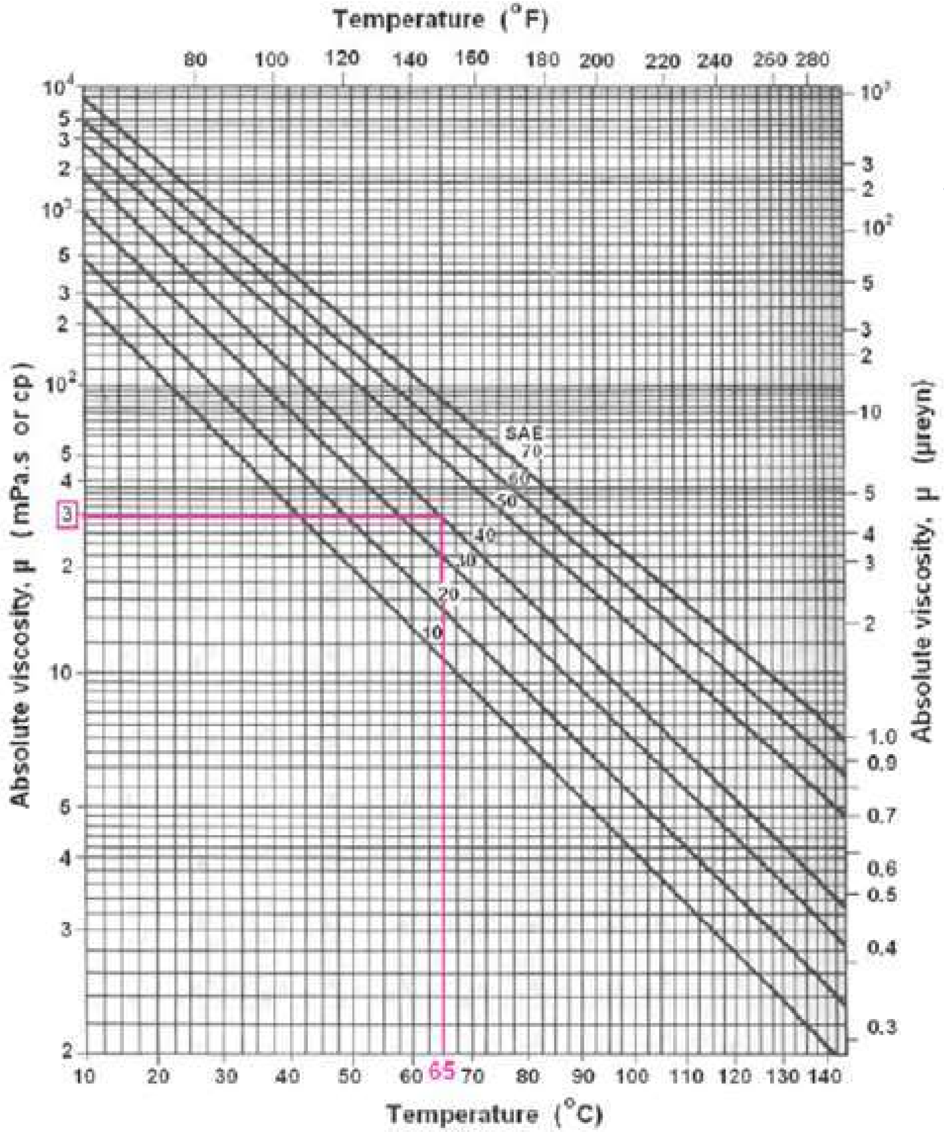
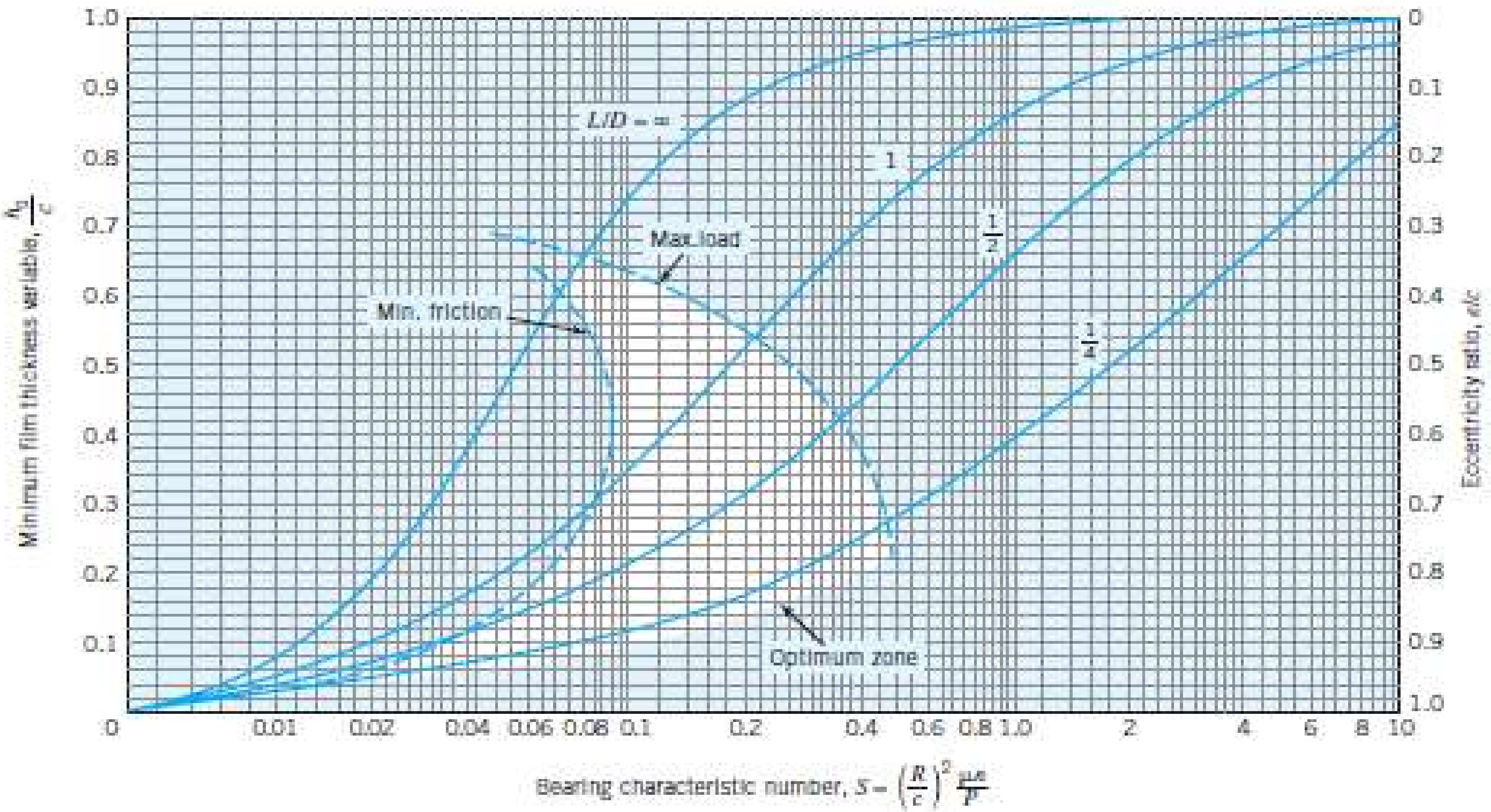


Chart for minimum-film-thickness variable

.



Coefficient of Friction

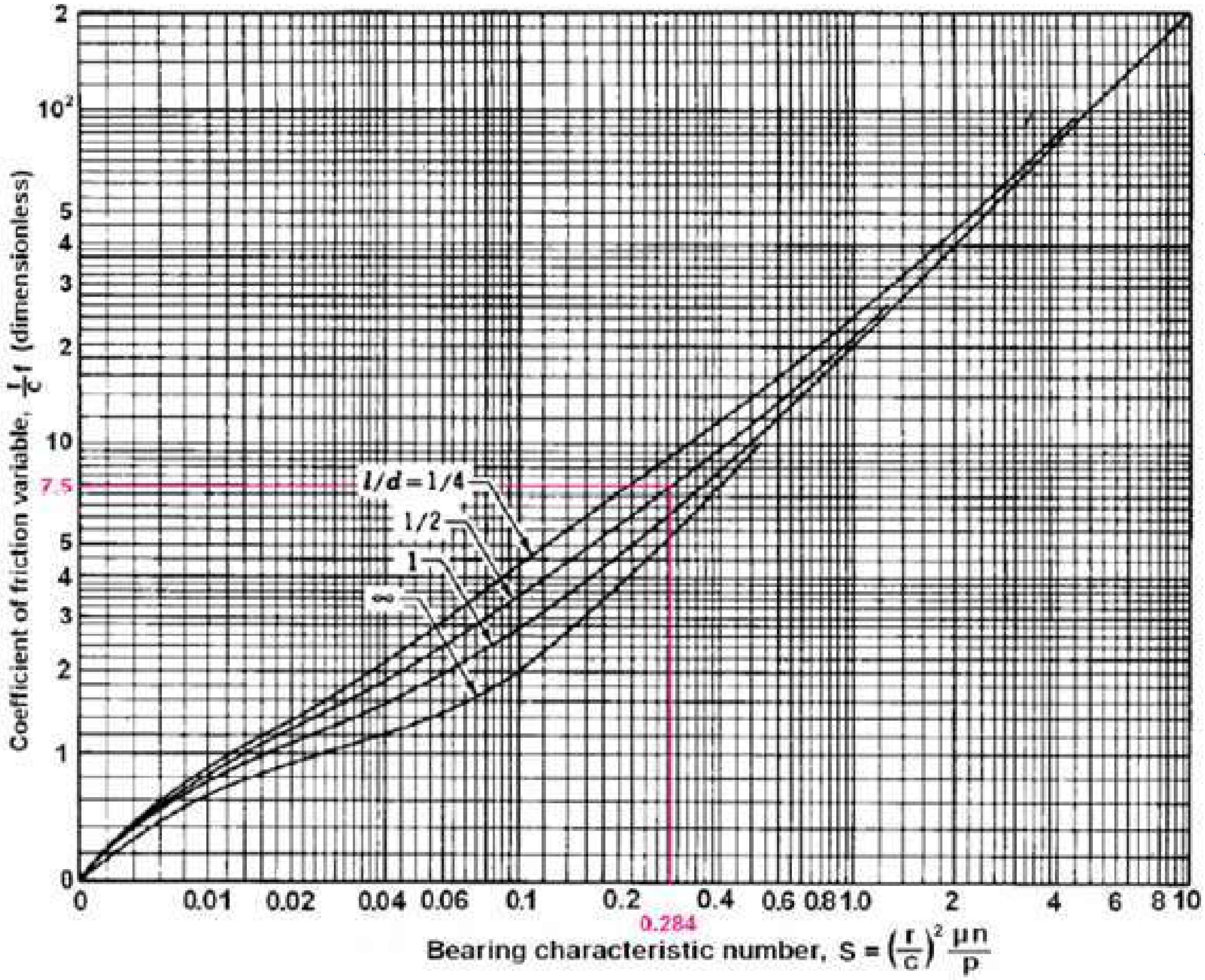


Chart for the position of minimum film thickness *ho*

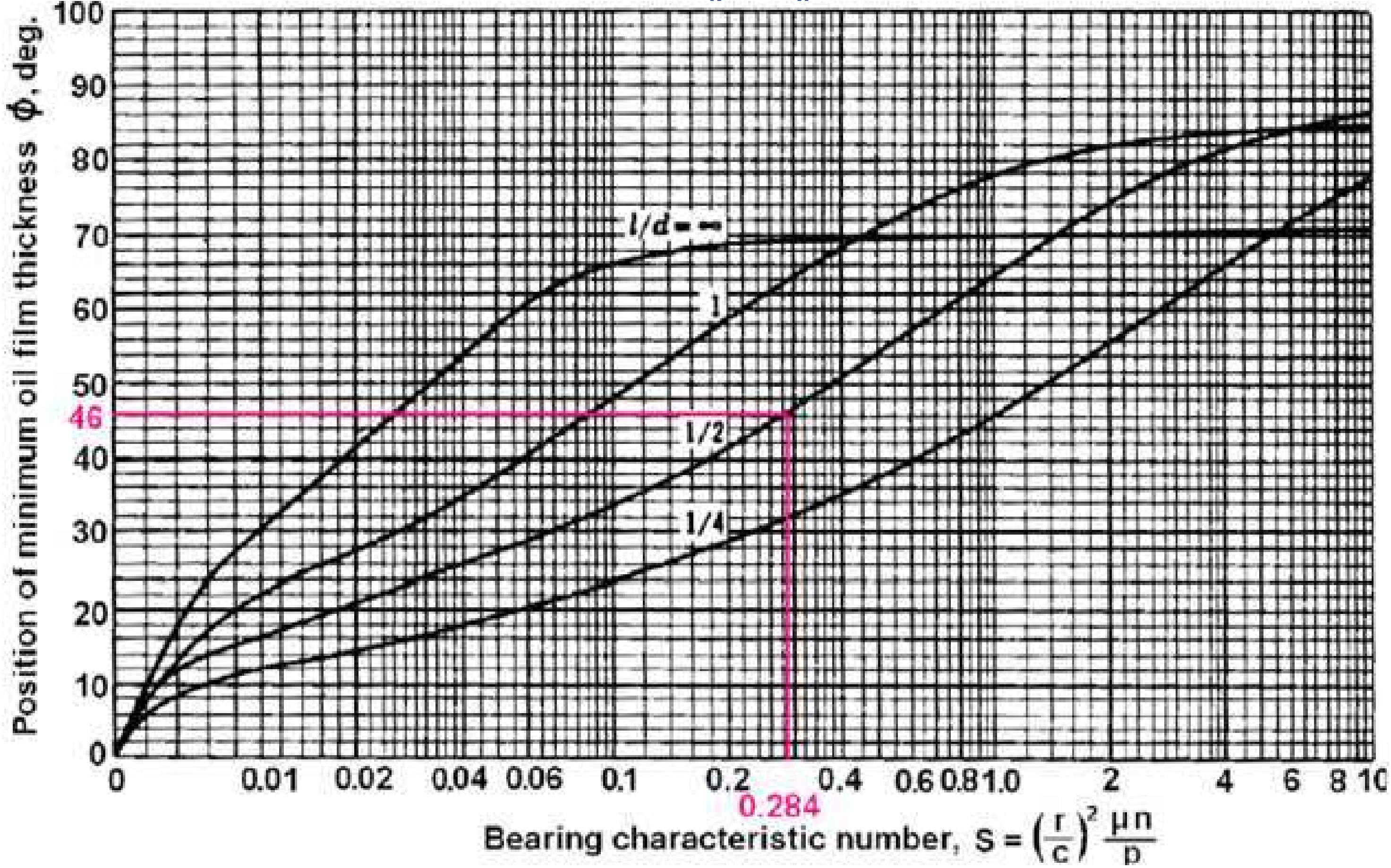


Chart for coefficient-of friction variable

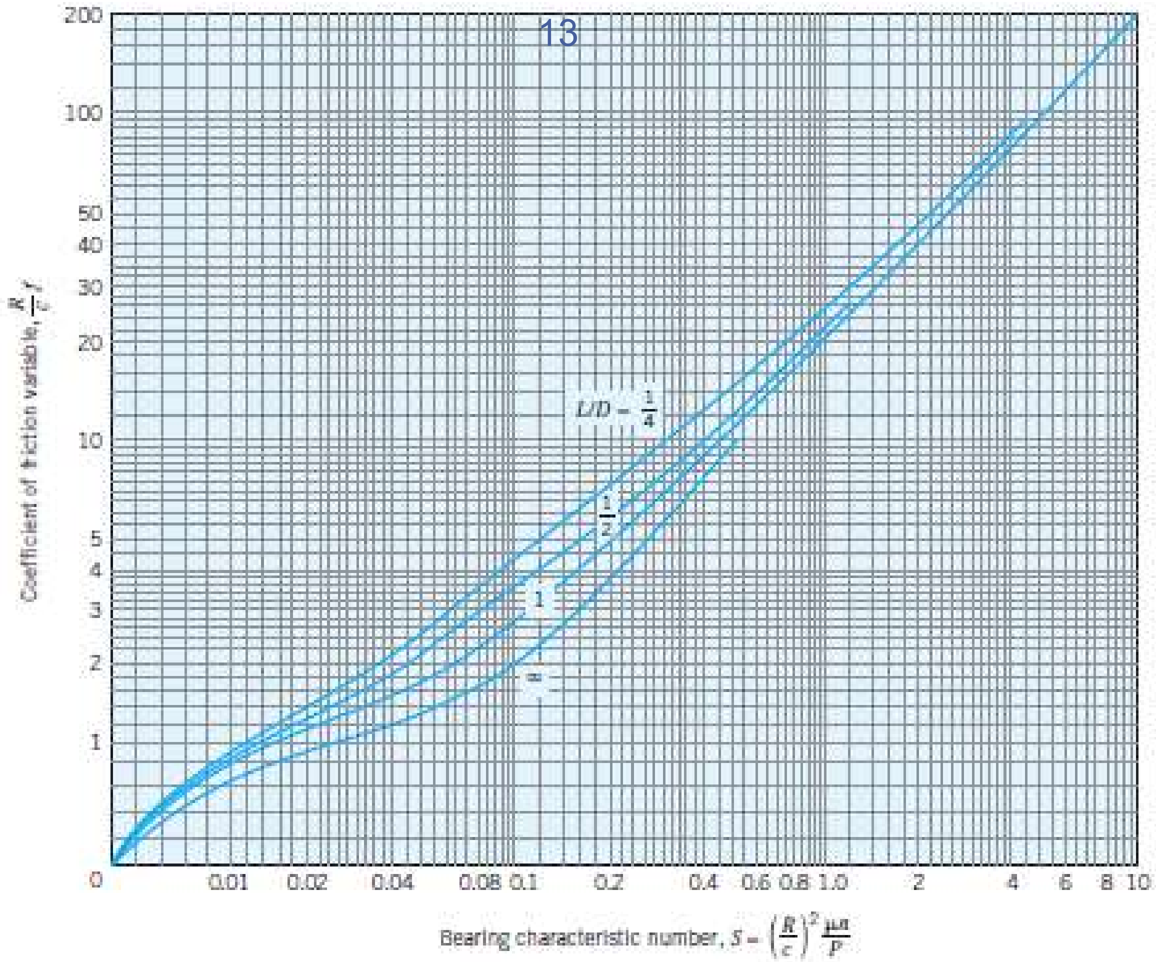


Chart for determining maximum film pressure

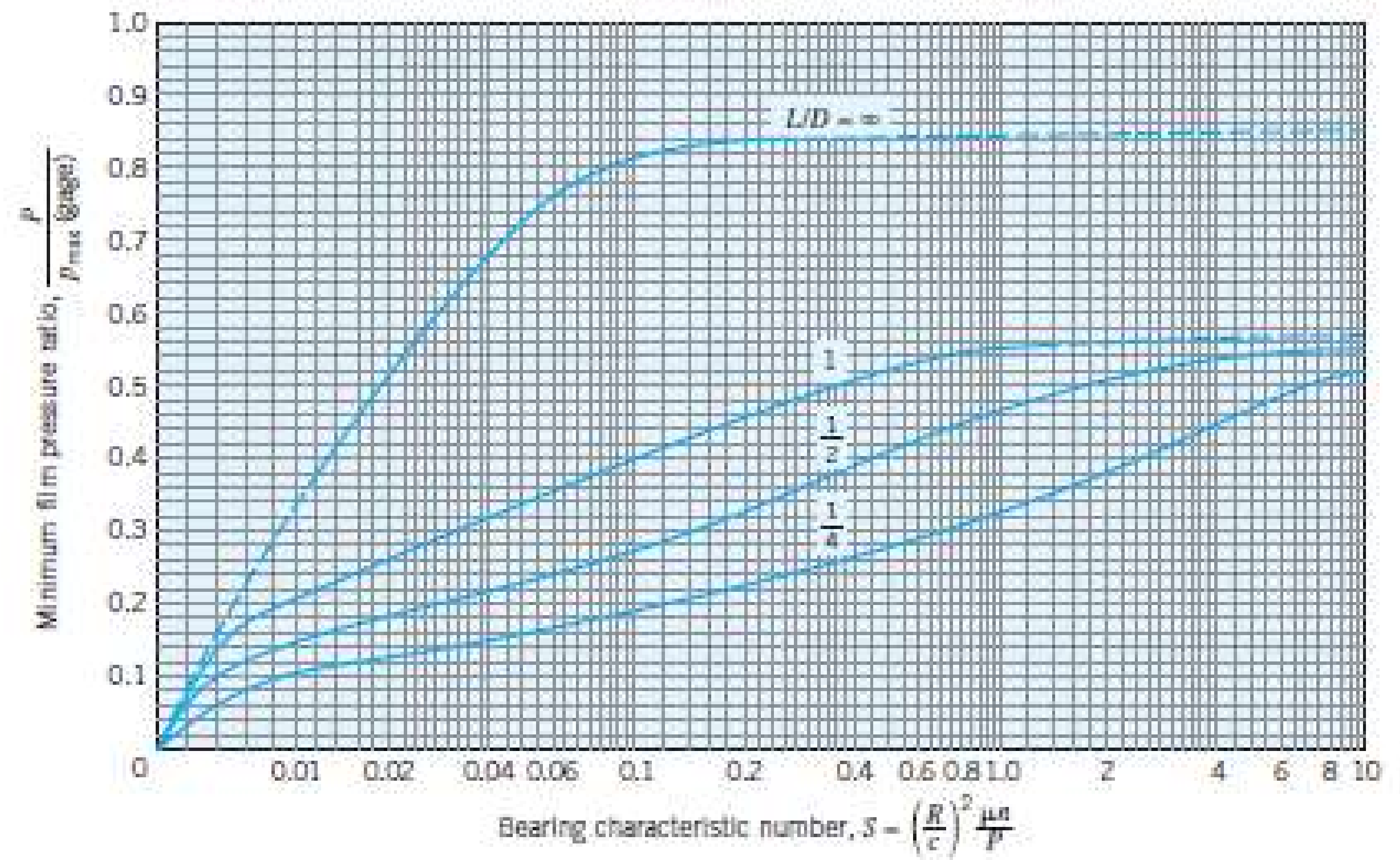


Chart for determining the position of the minimum film thickness *h*0

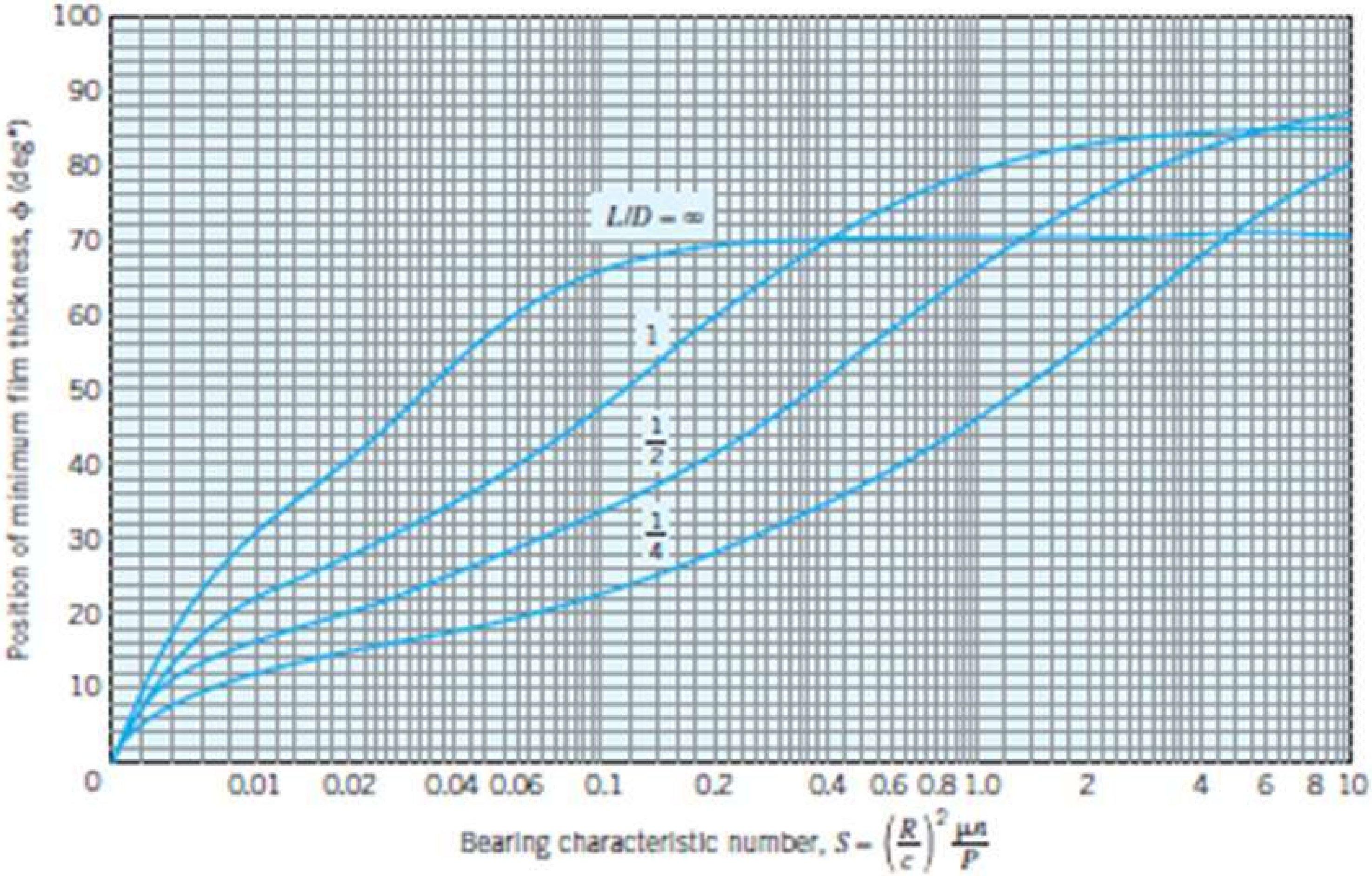


Chart for positions of maximum film pressure and film termination

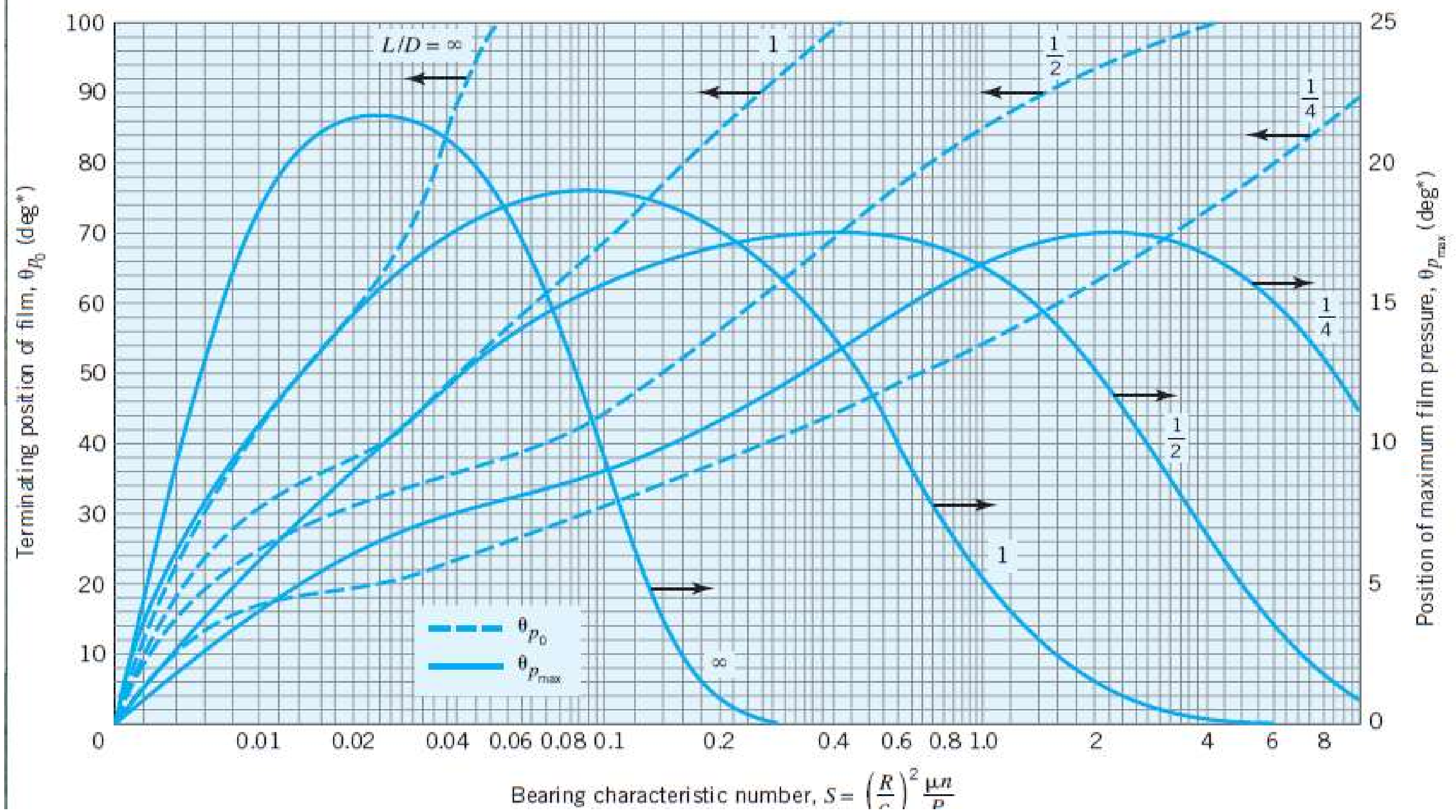


Chart for flow variable

17

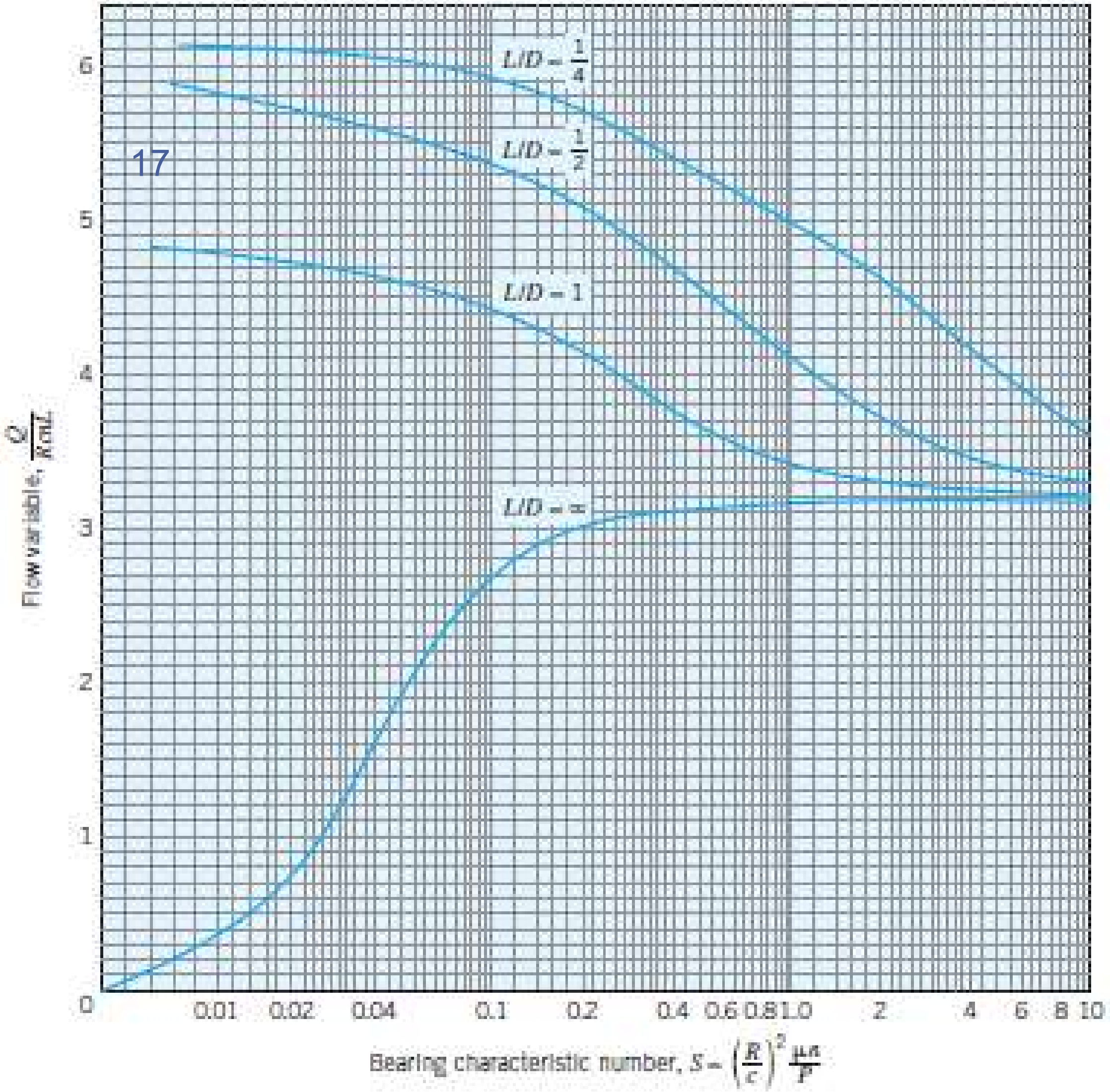


Chart for the ratio of side flow to total flow

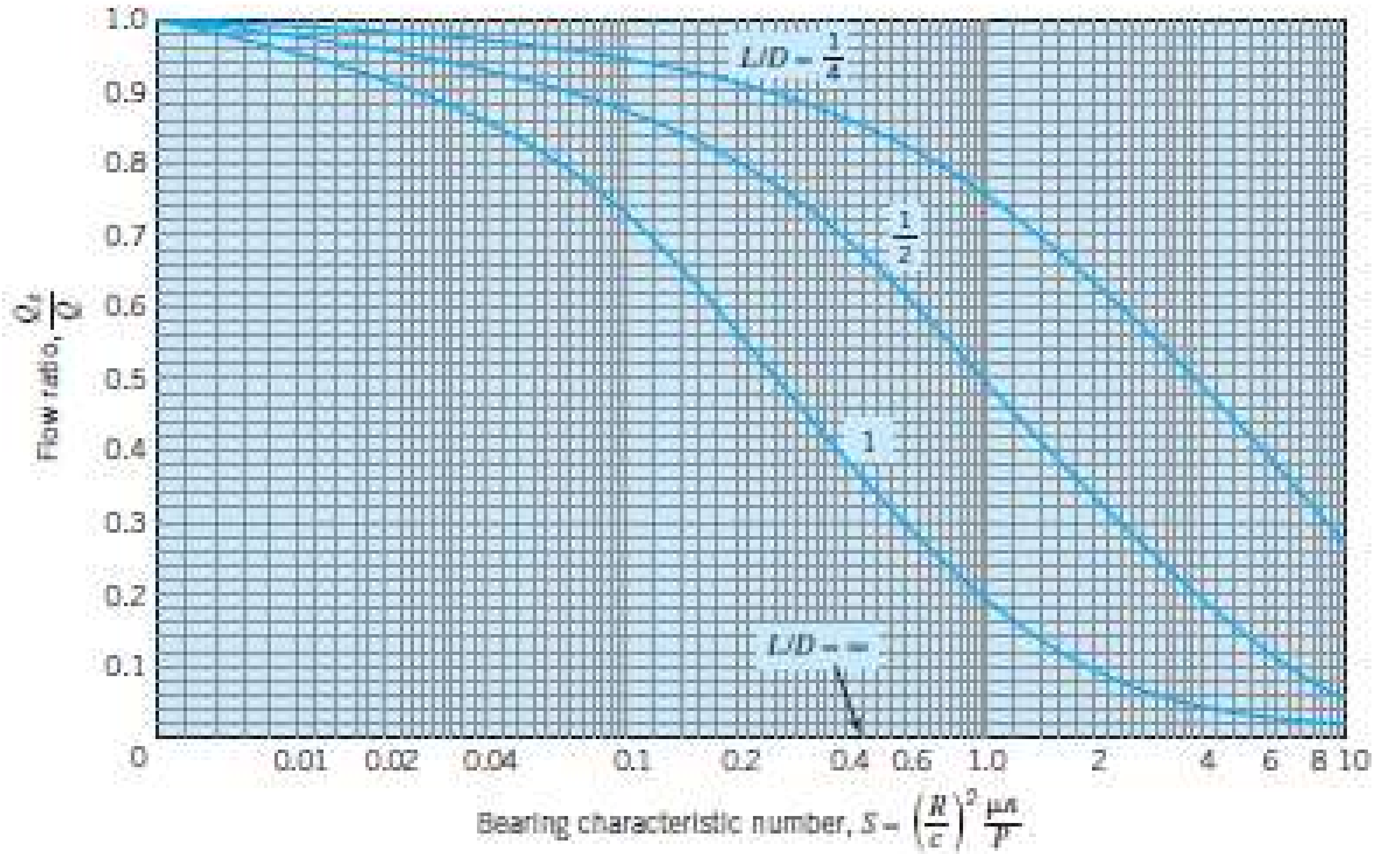
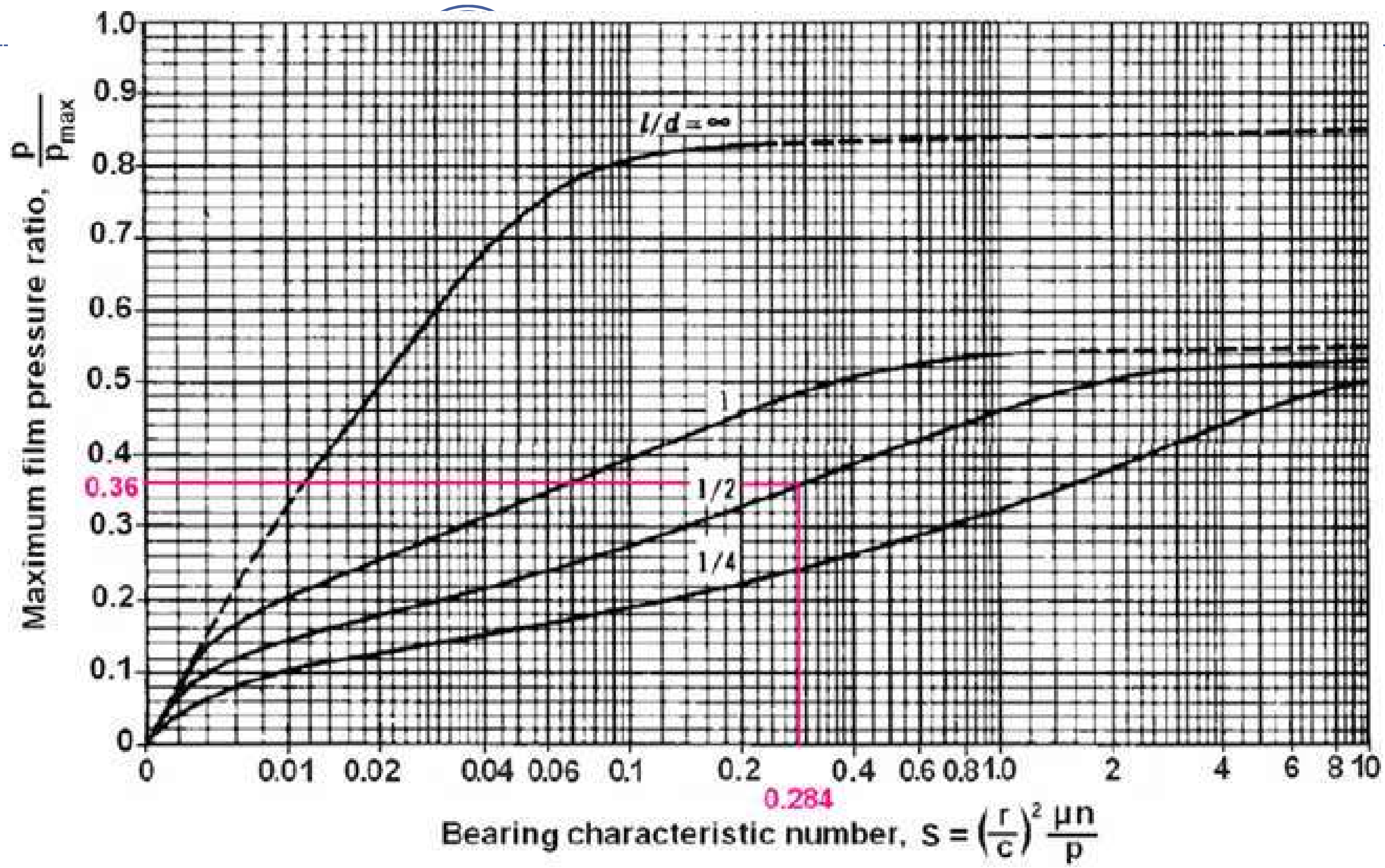
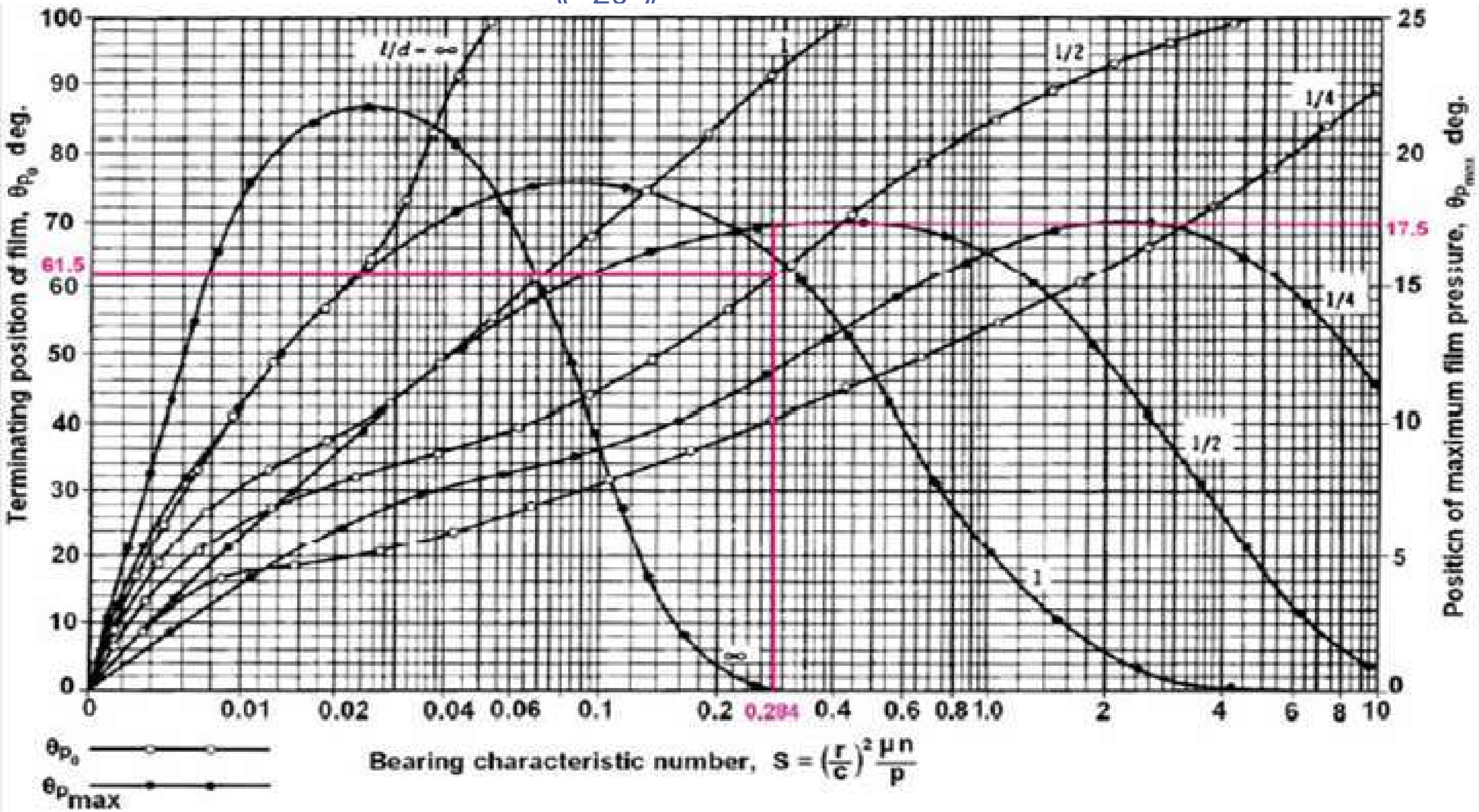


Chart for maximum film pressure

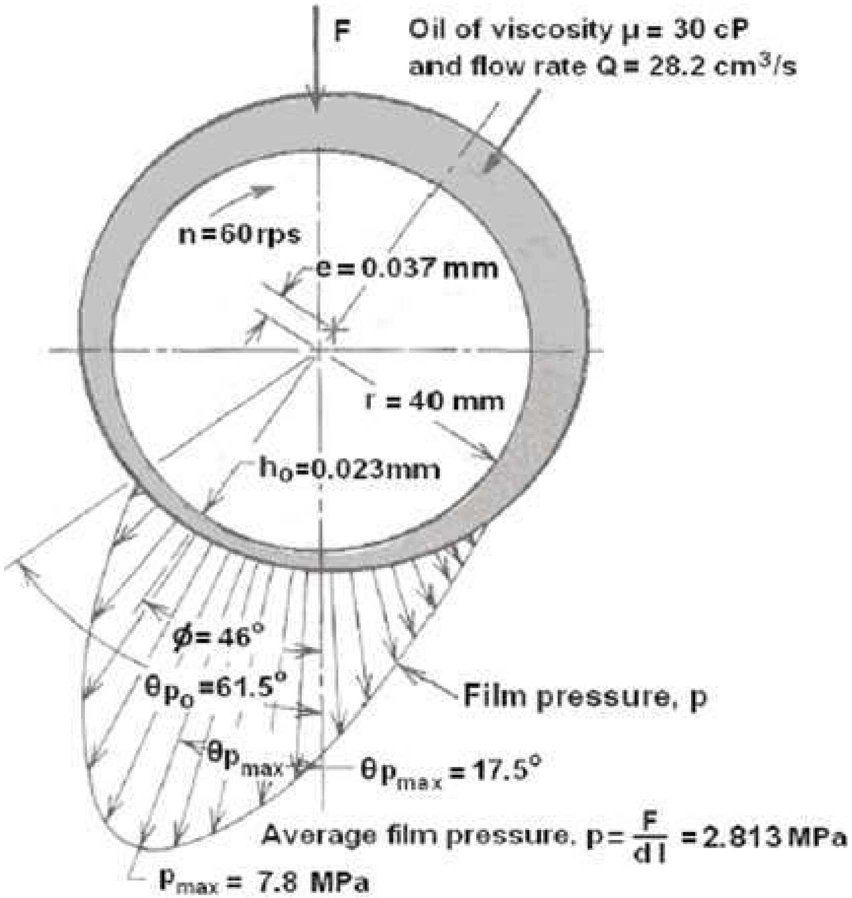


Terminating position of oil film and position of maximum film pressure



- **Fig – 15(a)**

Stable hydrodynamic Lubrication Diagram



- **Fig – 10(a)**

End of Lecture

Any Questions?

INTERACTION IS HIGHLY ENCOURAGED

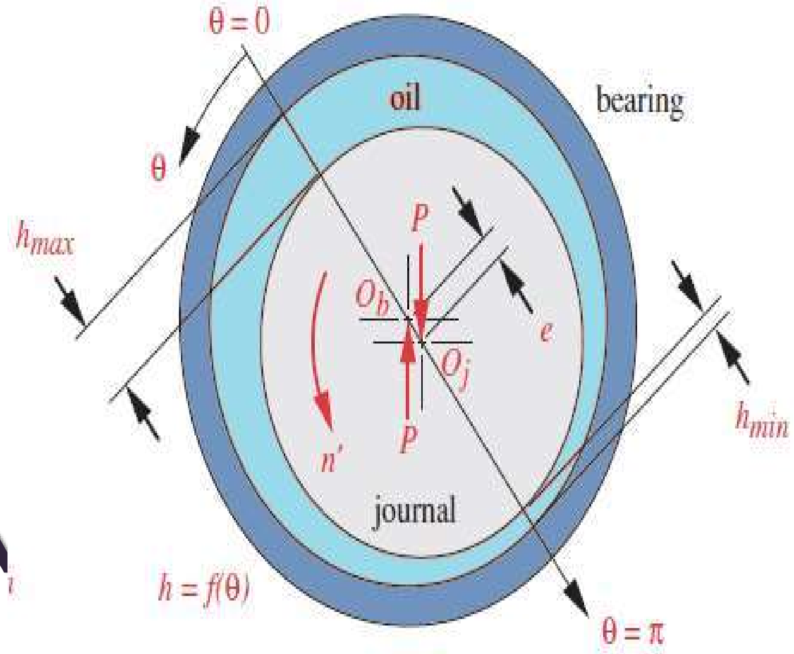
Lecture - 8

Hydrodynamic Lubrication

By

Prof. M. Naushad Alam

MACHINE DESIGN II
MEC 3110



MECHANICAL ENGINEERING DEPT.
A.M.U. ALIGARH

- **Heat dissipation and equilibrium oil temperature**

- The heat generated in the bearing should be effectively dissipated.
- This is required to attain equilibrium conditions in a short time.
- Average or equilibrium temperature of the oil should not exceed 93 to 123 oC to prevent quick deterioration of the oil.

Heat dissipation and equilibrium oil temperature

The frictional heat generated can be found from the load (W) coefficient of friction (f), and the journal speed (N).

$$\mathbf{v = 2\,\pi\,Nd\,/\,60{,}000\ \ (m/\,s)}$$

where ***N*** is in *rpm* & ***d*** in *mm*.

Frictional power loss:

$$\mathbf{H_G = Q\,f\,v} \qquad \mathbf{(3.1)}$$

where H_G is expressed in *Nm/s* or W

Heat dissipated:

$$\mathbf{H_d = C\,A\,(T_H - T_A)} \quad \text{in W or Nm/s} \qquad \mathbf{(3.3)}$$

where,

C = combined the heat transfer coefficient (radiation and convection) in $W/m^2.{}^oC$

T_H = surface temperature of the housing, ^oC

T_A = temperature of surrounding air, ^oC.

The value of C depends on the material, colour, geometry and roughness of the housing, temperature difference between the housing and surrounding objects and temperature and velocity of the air.

$C = 11.4\ W/m^2.{}^oC$	for still air
$C = 15.3\ W/m^2.{}^oC$	for average design practice
$C = 33.5\ W/m^2.{}^oC$	for air moving at 2.5 m/s

Heat dissipated, $\mathbf{H_d = C\ A\ (T_H - T_A)}$ **(3.3)**

Where, A, the exposed surface area of the housing may be roughly estimated as a function of the projected area of the bearing.

$$A = C_h\,(A_{proj.})$$

$$= C_h.\pi dl$$

$$= 20\ dl$$

C_h is called surface area factor,

it ranges from 5 to 15, depending upon design considerations.

- An expression similar to eqn. (3.3) can be written between the temperature difference $T_O - T_H$ between the lubricant oil film and the housing.
- The relationship depends on the lubrication system and the quality of lubricant circulation. Oil bath lubrication system in which a part of the journal is immersed in the lubricant provides good circulation.
- A ring oiled bearing in which oil rings ride on top of the journal or an integral collar on journal dip into the oil sump and provides fair circulation for many purposes.
- Wick feeding will result in inadequate circulation and should be limited to very light load application and is not considered here.

$$T_O - T_H = b\,(T_H - T_A) \qquad (3.4)$$

where T_O is the average oil film temperature and b is a constant depending on lubrication system.

Heat dissipated: $H_d = C\,A\,(T_H - T_A)$ (3.3)

$$T_O - T_H = b\,(T_H - T_A) \qquad (3.4)$$

Since T_O and T_A are known, combining eqn. (3.3) & (3.4),

$$H_d = C\,A\left(\frac{1}{b+1}\right)(T_o - T_A) \qquad (3.5)$$

$$H_d = C\,A\,B\,(T_O - T_A) \qquad (3.6)$$

Where B = 1/ (b+1) and a rough estimate of this is given in Table 3.1.

In heat balance computation, the oil film temperature and hence the viscosity of the lubricant in a self contained bearing are unknown. The determination is based on iterative process where the heat generated and heat dissipated match giving the equilibrium temperature. This is a time involving procedure.

Table 3.1 Value of the constant B

Lubrication system	Condition	Range of B
Oil ring	Moving air	0.333 - 0.500
Oil ring	Still air	0.667 – 0.500
Oil bath	Moving air	0.667 – 0.500
Oil bath	Still air	0.714 -0.833

Temp rise from Chart

The oil temperature rise can be estimated from:

The chart given by Raimondi and Boyd

or from the heat balance equation in the case of self contained bearings as in the case of ring, collar or oil bath lubrication.

$$T_{var} = \gamma C_H \left(\frac{\Delta T}{p} \right) \qquad (3.2)$$

Where γ is the density of the oil 861 kg /m^3

C_H is the specific heat of the oil, an average value of 1760 J/ kg. °C may be taken.

ΔT is the temperature rise °C and P is the film pressure in Pa.

Numerical Problem

A journal of a stationary oil engine is 80 mm in diameter and 40 mm long. The radial clearance is 0.060mm. It supports a load of 9 kN when the shaft is rotating at 3600 rpm with SAE 40 oil supplied at atmospheric pressure and assume average operating temperature is about 65oC as first trial for inlet oil temperature of 45oC. Using Raimondi-Boyd charts analyze the bearing temperature under steady state operating condition.

Solution

Data given:

$d = 80$ mm; $l = 40$ mm;

$c = 0.06$ mm; $F = 9$kN;

$N = 3600$rpm $= 60$ rps;

SAE 40 oil; $T_o = 65$ oC; $T_i = 45$ oC.

$$p = F / l\,d$$

$$= 9 \times 1000 / 80 \times 40$$

$$= 2.813 \text{ MPa}$$

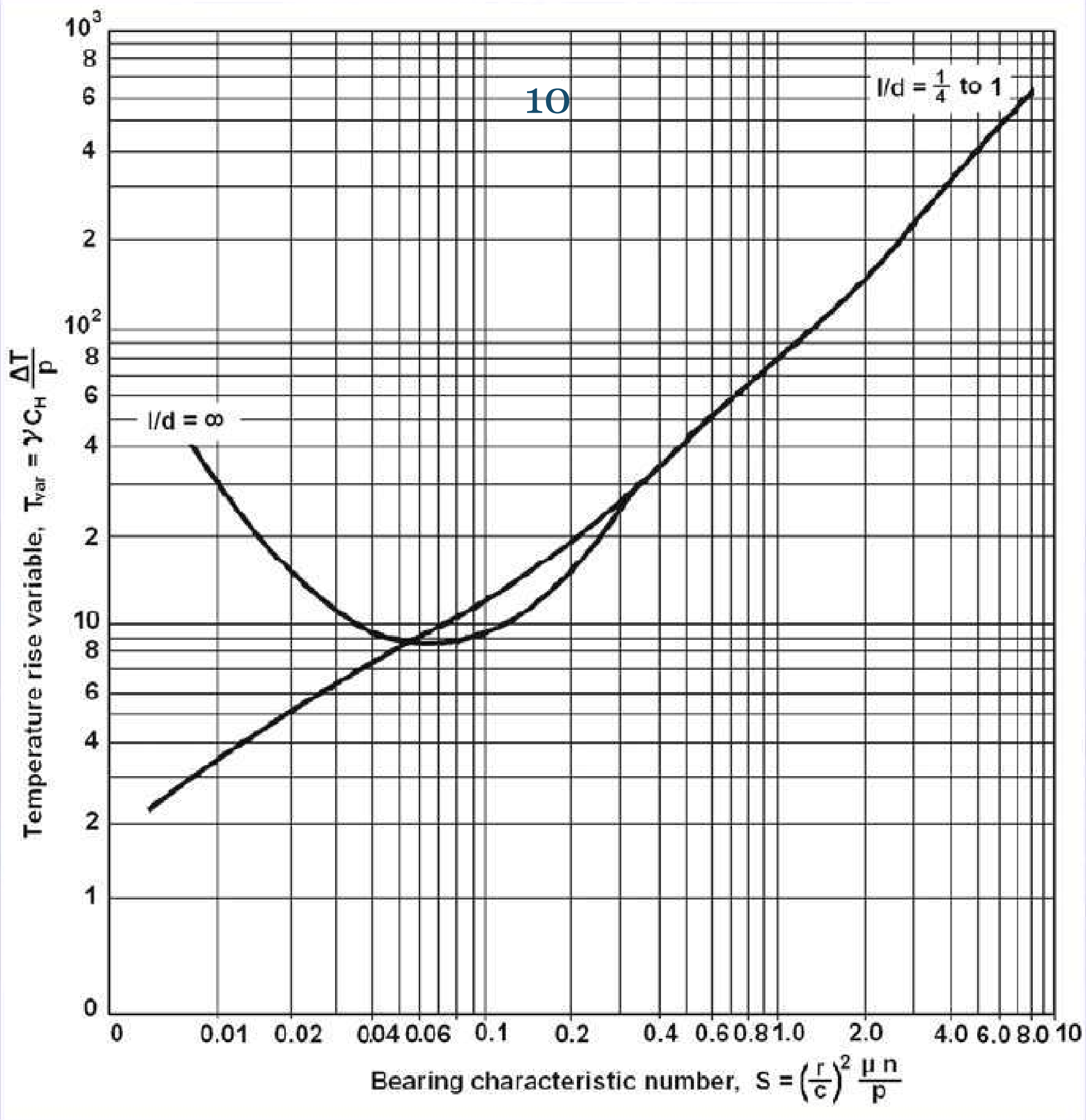


Fig.3. 5 Chart for temperature variable, $T_{var} = \gamma\ C_H\ (\Delta T/p)$

$T_{av} = T_i + 0.5\,\Delta T$

$= 45 + 0.5 \times 46 = 68^{\circ}C$

From the chart 2.3(b)

At $T_{av} = 68^{\circ}C$,

$\mu = 26$ Pa.s

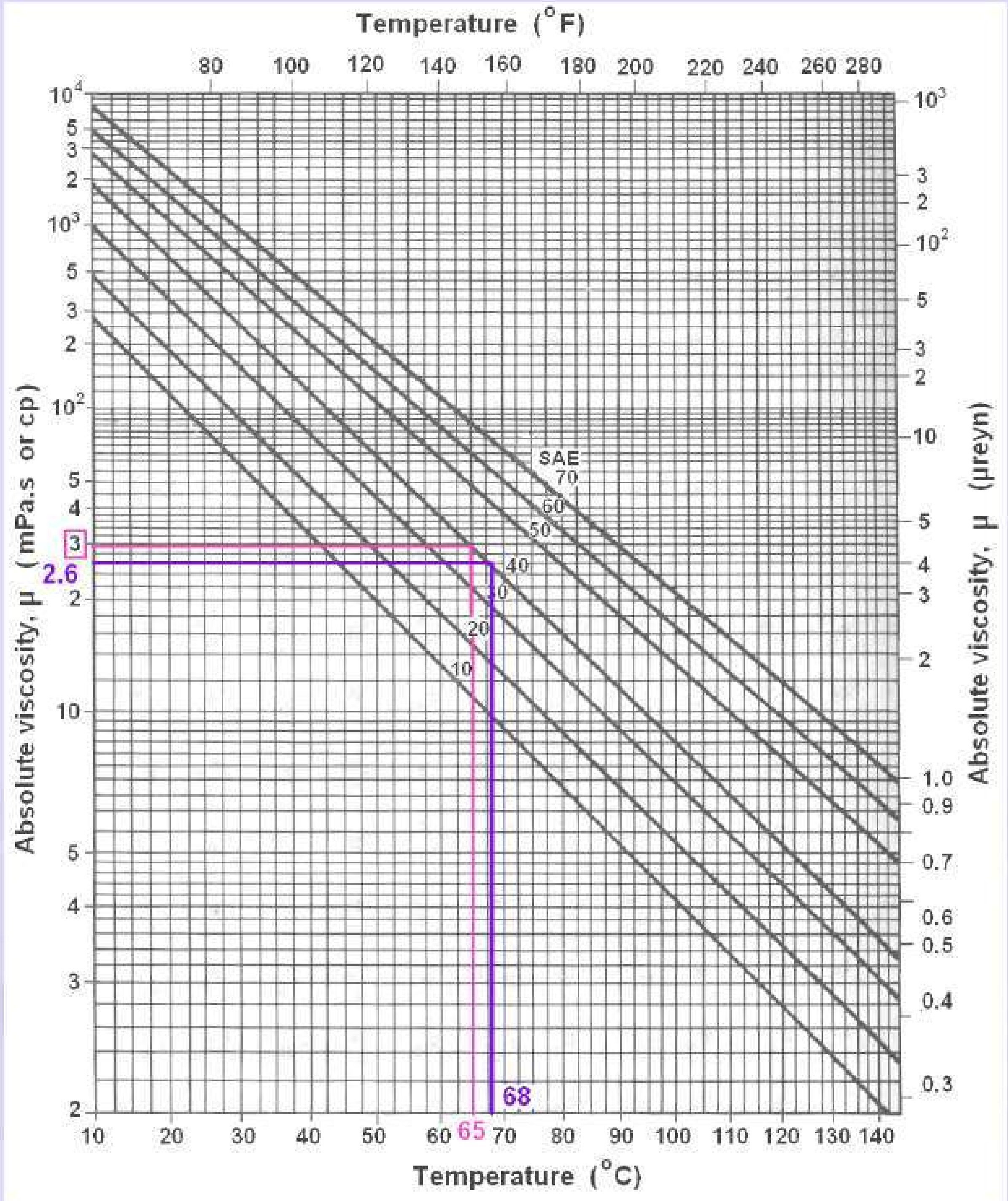


Fig. 2.3b Viscosity – temperature curves of SAE graded oils

From the graph 2.3(a) we obtain

μ = 30 cP at 65 oC

for SAE 40 oil

$$S = \left(\frac{r}{c}\right)^2 \left(\frac{\mu n}{p}\right)$$

$$= \left(\frac{40}{0.06}\right)^2 \left(\frac{30x10^{-3} \text{ x } 60}{2.813 \text{ x } 10^6}\right)$$

$$= 0.284$$

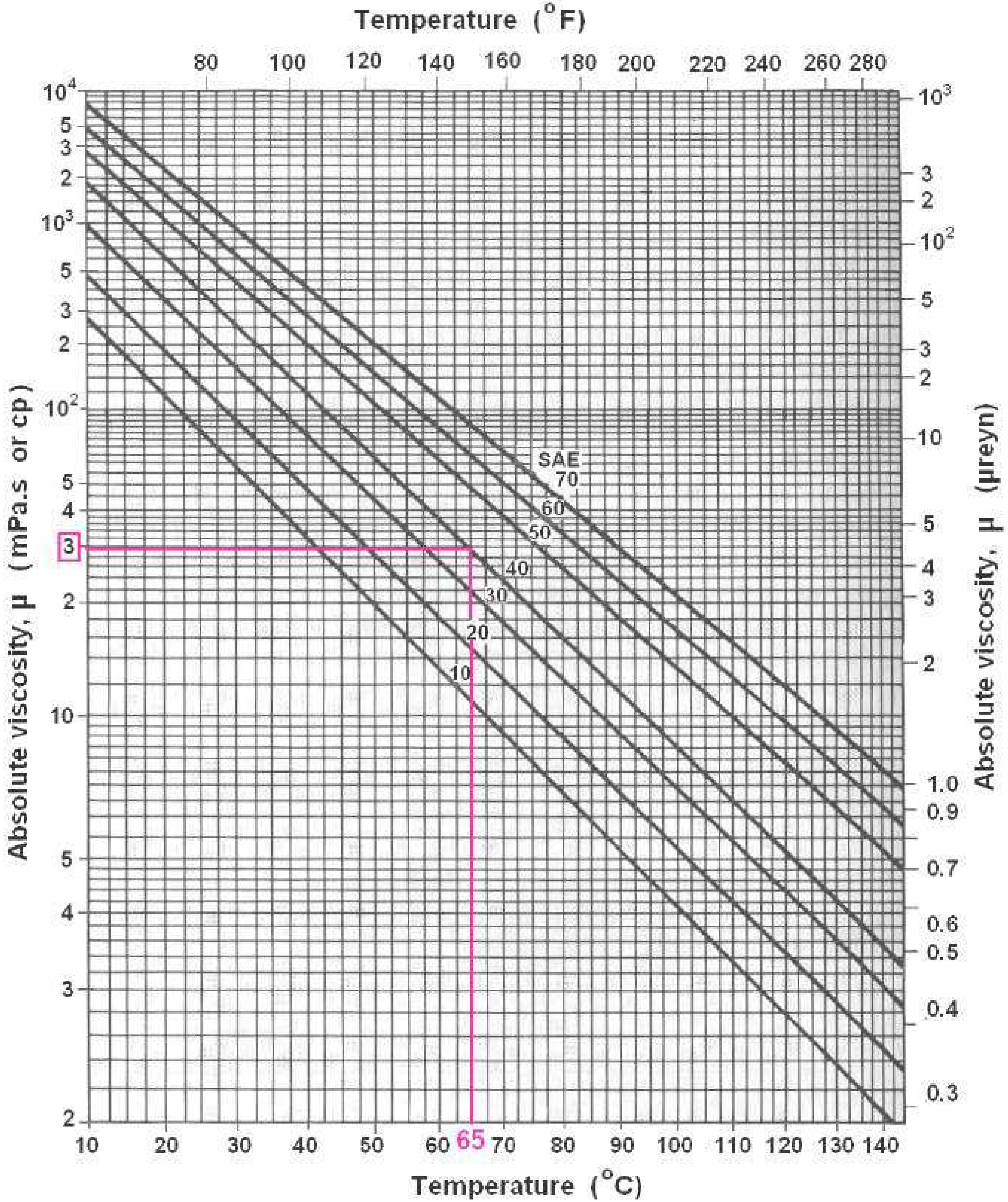


Fig. 2.3(a) Viscosity – temperature curves of SAE graded oils

From the chart 3.5(a):

For S = 0.284 and l /d = ½,

T_{var} = 25

From the previous equatio

$$T_{var} = \gamma C_H \left(\frac{\Delta T}{p}\right)$$

$$\Delta T = \frac{T_{var}\, p}{\gamma\, C_H}$$

$$= \frac{25 \times 2.813 \times 10^6}{861 \times 1760}$$

$$= 46^\circ C$$

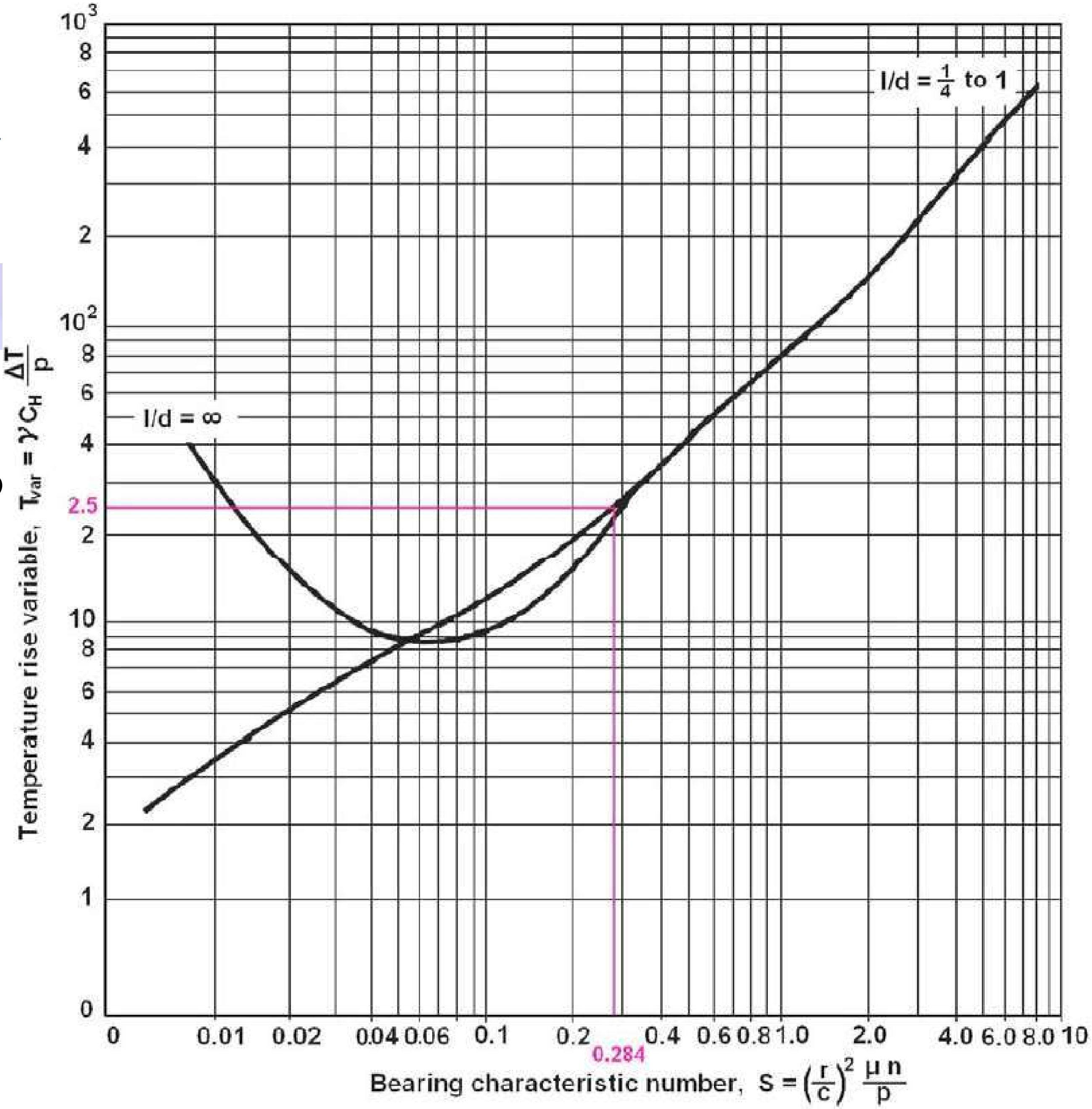


Fig. 3.5(a) Chart for temperature variable, $T_{var} = \gamma\, C_H\, (\Delta T/p)$

…

From Fig. 3.5 (b),
for S = 0.246,
T_{var} = 22.5,

calculated value of ΔT = 41.4 oC

$T_{av} = T_i + 0.5\,\Delta T$
$= 45 + 0.5 \times 41.4$
$= 65.7$oC.

Hence equilibrium temperature will be about 66 oC.

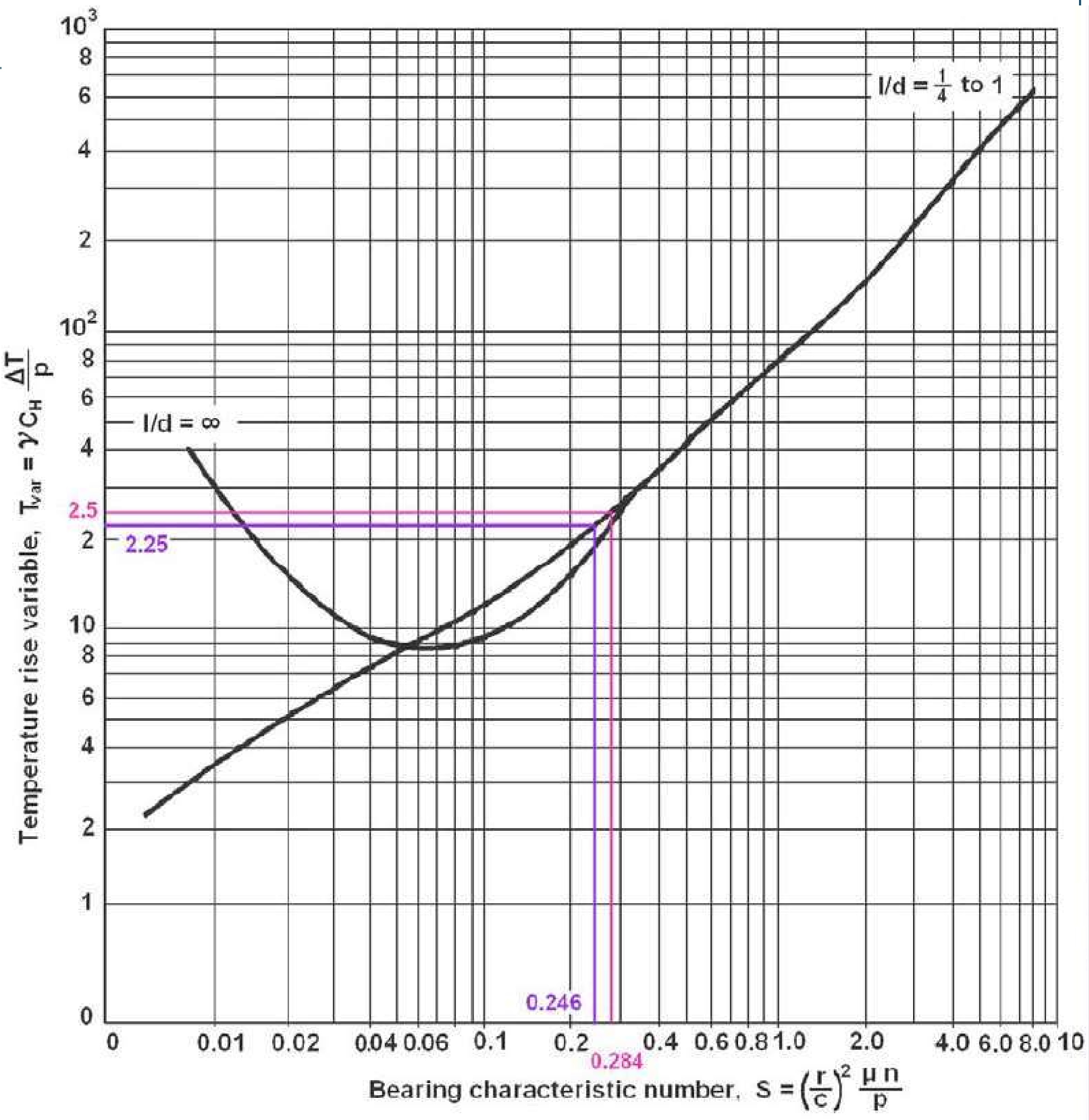


Fig. 3.5(b) Chart for temperature variable, $T_{var} = \gamma C_H (\Delta T/p)$

Problem –2

A sleeve bearing is 40 mm in diameter. and has a length of 20 mm. The clearance ratio is 1000, load is 2.5 kN, and journal speed is 1200 rpm. The bearing is supplied with SAE 30 oil. The ambient temperature is 35 oC. Determine the average oil film temperature in equilibrium condition, assuming that the bearing is lubricated by an oil bath in moving air.

Data: d = 40 mm; l = 20 mm; r/c = 1000; SAE 30 Oil;

$T_A = 35^oC$; Lubrication is by oil bath in still air.

Analysis:

1. $p = F/d\,l = 2.5 \times 10^3 / 0.04 \times 0.02 = 3.13 \times 10^6$ Pa

2. Expecting the oil average temperature to be 60^oC

$\mu = 26.5$ cP or mPa.s for SAE 30 oil. From Fig.2.3c

3. n = 600/60 = 10 rps.

4. $S = \left(\frac{r}{c}\right)^2 \left(\frac{\mu n}{p}\right) = 1000^2 \left(\frac{26.5 \times 10^{-3} \times 10}{3.13 \times 10^6}\right) = 0.085$

5. $\frac{r}{c}f = 3.05$ for S = 0.085 and (l /d) = 0.5 from Fig.2.11b.

6. $f = 3.05\frac{c}{r} = 3.05x10^{-3} = 0.00305$

7. $v = \frac{\pi dn}{60000} = \frac{\pi x 40 x 600}{60000} = 1.26 m/s$

8. $H_g = F\ f\ v = 2500\ x0.00305\ x\ 1.26 = 9.61\ Nm/s$

9. $H_d = CAB\ (T_o - T_A) = H_g$ from which

$C = 33.5\ W/m^2.^{\circ}C$ for moving air

B = 0.667 from Table 2.1 for oil bath in moving air.

$A = 20\ d\ l = 20\ x\ 0.04\ x\ 0.02 = 0.016\ m^2$

10. $T_o = T_A + H_g / CAB$

$= 35 + 9.61 / (33.5x0.016\ x0.667)$

$= 61.9^{\circ}C$

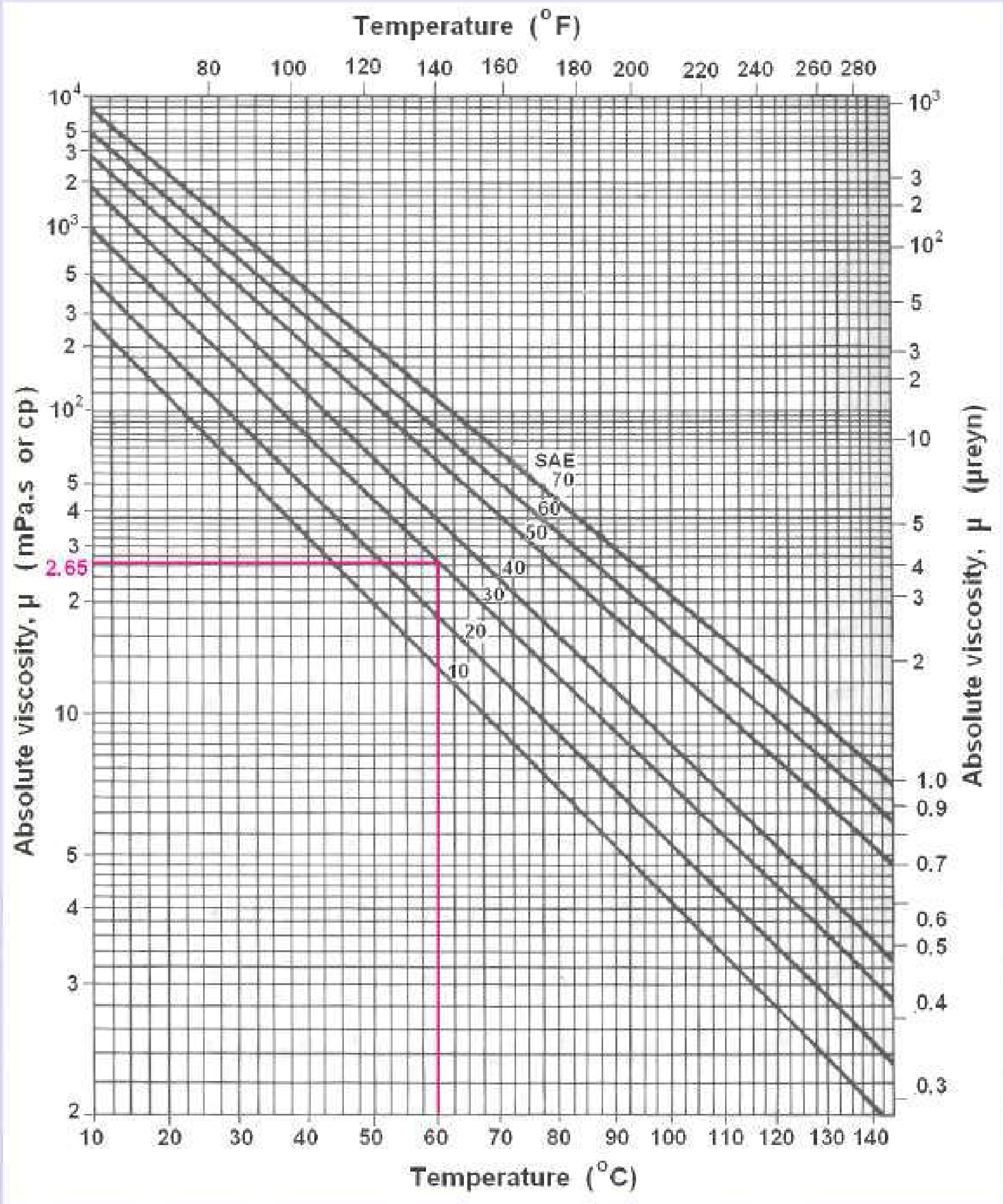


Fig.2.3c Viscosity – temperature curves of SAE graded oils

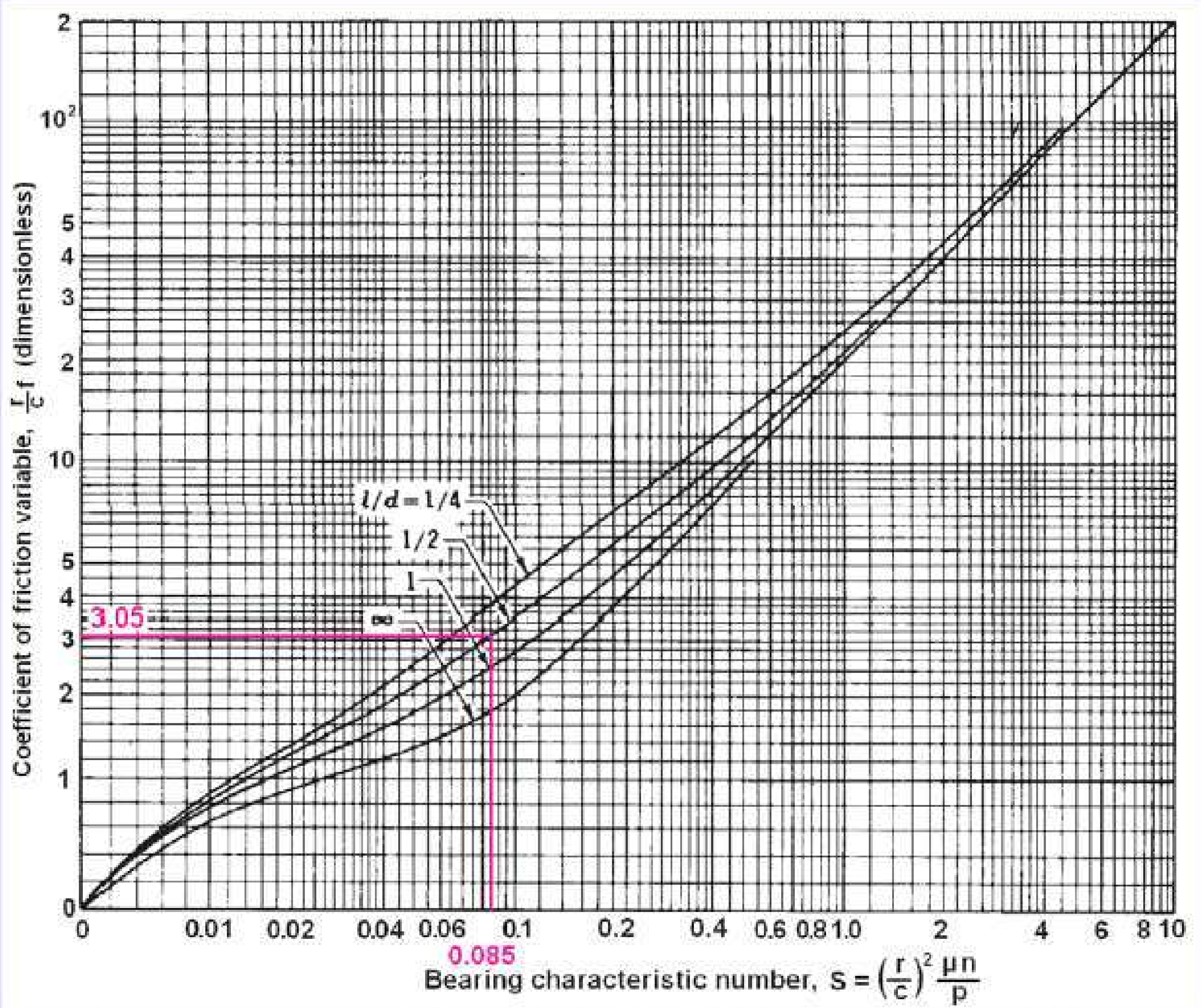


Fig. 2.11b Chart for coefficient of friction variable

Iteration 2

1. For oil temperature of 61.9°C, μ = 26.5 mPa.s for SAE 30 oil from Fig.2.3d

2. $S = \left(\frac{r}{c}\right)^2 \left(\frac{\mu n}{p}\right) = 1000^2 \left(\frac{24.5 \times 10^{-3} \times 10}{3.13 \times 10^6}\right) = 0.078$

3. f = 0.00285

4. H_g = F f v = 2500 x0.00285 x 1.26 = 8.98 Nm /s

5. $T_o = T_A + H_g$ / CAB

   = 35 + 8.98 / (33.5x0.016 x0.667)

   = 60.1°C

6. Hence the equilibrium temperature of oil will be around 60.1°C.

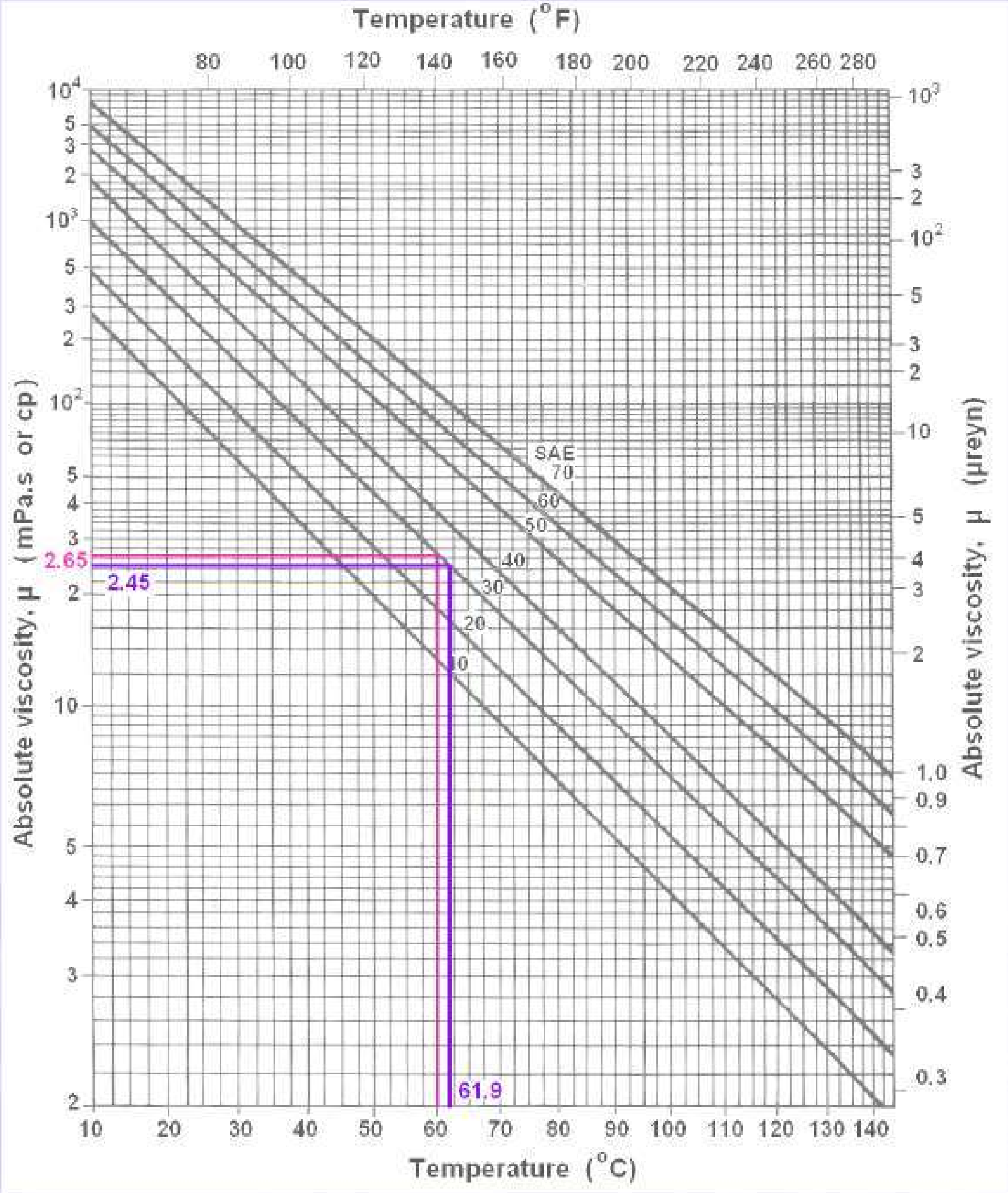


Fig.2.3d Viscosity – temperature curves of SAE graded oils

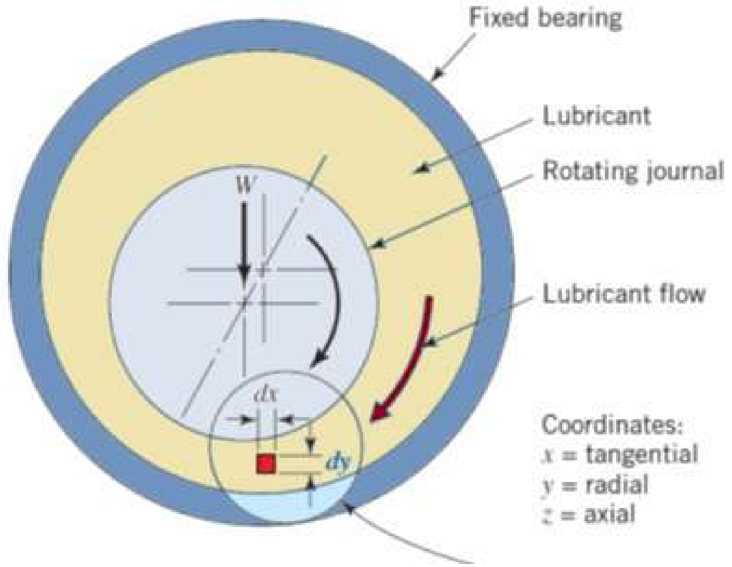
Lecture - 9

Hydrodynamic Lubrication

By

Prof. M. Naushad Alam

MACHINE DESIGN II
MEC 3110



MECHANICAL ENGINEERING DEPT.
A.M.U. ALIGARH

HYDRODYNAMIC LUBRICATED JOURNAL BEARING DESIGN

The design procedure of hydrodynamic bearing is very elaborated one with theory and practice being judiciously blended together.

1 Unit loading

The load per unit journal projected area is denoted by **p**.

In many applications like engine bearings, momentary peak loads result in bearing pressures of the order of ten times the steady state values.

The hydrodynamic bearings can take up such peak loads without any problem.

The recommended values of steady unit load for various applications are given in Table 3.1.

This helps in selecting suitable diameter for any particular the application.

Unit loads for journal bearings (Table 3.1)

(a) Relatively Steady Loads $p = F_{max} / d\,l$

Applications	Unit loads MPa	Applications	Unit loads MPa
Electric motors	0.8 – 1.5	Air compressors Main bearing	1.0 - 2.0
Steam turbines	1.0 – 2.0	Air compressors Crank pin bearing	2.0 – 4.0
Gear reducers	0.8 – 1.5	Centrifugal pumps	0.6 – 1.2

(b) Relatively Fluctuating Loads $p = F_{max} / d\,l$

Applications	Unit loads MPa	Applications	Unit loads MPa
Diesel Engines		Automotive gasoline engines	
Main bearings	6 – 12	Main bearings	4 - 5
Connecting rod bearings	8 – 15	Connecting rod bearings	10 – 15

Bearing l / d ratios

- Ratios - 0.25 to 0.75 are now commonly used in modern machinery whereas in older machinery closer to unity was used.
- Longer bearings have less end leakage and reduced oil flow requirements and high oil temperature.
- Short bearings are less prone to edge loading from shaft deflection and misalignments, need higher flow rate and run cooler.
- The shaft size is found from fatigue strength and rigidity considerations.
- Bearing length is found from permissible unit loads.

Acceptable values of h_o:

- The minimum acceptable oil film thickness, ho, depends on surface finish. Trumpler suggests the relationship

$$h_o \geq 0.005 + 0.00004\, d \text{ (units in mm) } (3.7)$$

- This equation applies only to bearings that have finely ground journal with surface roughness not exceeding 5μm, that have good standards of geometric accuracy circumferential out of roundness, axial taper, and "waviness" both circumferential and axial; and that have good standards of oil cleanliness.

- A factor of safety of 2 is suggested for steady loads that can be assessed with good accuracy.

Clearance ratios c/r

- For journals 25 to 150 mm in diameter and for precision bearings (c / r) ratio of the order 0.001 is recommended.
- For less precise bearings of general machinery bearings (c / r) ratio up to about 0.002 is used.

Table 3.3 Clearance ratio: $\psi = c/r$ in 10^{-3}

Working pressure p MPa	Peripheral speed m/		
	Low <2	Medium -2 to 3	High >3
Low to medium p<8	0.7 – 1.2	1.4-2.0	2-3
High p>8	0.3-0.6	0.8-1.4	1.5-2.5

Clearance ratios c/r

- For rough-service machinery (c / r) ratio of 0.004 is used.
- In any specific design the clearance ratio has a range of values, depending on the tolerances assigned to the journal and bearing diameter.

Table 3.3 Clearance ratio: $\psi = c/r$ in 10^{-3}

Working pressure p MPa	Peripheral speed m/		
	Low <2	Medium -2 to 3	High >3
Low to medium p<8	0.7 – 1.2	1.4-2.0	2-3
High p>8	0.3-0.6	0.8-1.4	1.5-2.5

Table 3.4. Surface roughness values R_1 and R_2 in µm (peak to valley height of shaft and bearing surface roughness)

Type of machining	Roughness values	Type of machining	Roughness values
Rough turning finish	16 - 40	Fine turning, reaming, grinding, broaching finish	2.5 – 6.0
Medium turning finish	6 - 16	Very fine grinding, lapping, honing	1 – 2.5

Important factors to be taken into account for designing a hydrodynamic bearing

1. The minimum oil film thickness to ensure thick film lubrication is given as ho ≥ 0.005 + 0.00004 d

2. Friction should be as low as possible to reduce the power loss ensuring adequate oil film thickness. Operation in the optimum zone in Raimondi chart ensures good design.

3. Ensure adequate supply of clean and cool oil at the bearing inlet.

4. Ensure that the oil temperature never exceeds 93 oC for long life of the oil.

Important factors for hydrodynamic bearing design

5. ***Grooves*** are to be provided for distribution of oil admitted to the bearing over its full length. If so, they should be kept away from highly loaded areas.

6. Choose a bearing material with enough strength at operating temperatures, adequate conformability and embeddability, and sufficient corrosion resistance.

7. Shaft misalignment and deflection should not be excessive.

8. Check the bearing loads and elapsed times during start-up and shutdown. Bearing pressures should be below 2MPa during these periods.

9. To arrive at a good design, right combinations of clearance and oil viscosity for given operating condition should be chosen. This will ensure running of the bearing with minimum friction and wear, and lowest possible temperature by dissipating the heat.

Problem 1

- A journal bearing of a centrifugal pump running at 1740 rpm has to support a steady load of 8kN. The journal diameter from trial calculation is found to be 120 mm. Design suitable journal bearing for the pump to operate under hydrodynamic condition.

Given Data:

$n = 1740$ rpm $= 29$ rps

$F = 8$ kN $= 8000$ N

$r = 0.5d = 60$mm

Problem 1

Given Data:

n = 1740 rpm = 29 rps

F = 8 kN = 8000 N, r = 0.5d= 60mm

From the Table, for centrifugal pumps, recommended unit load is 0.6 to 1.2MPa.

(a)Relatively steady loads $p = F_{max} / d\,l$

Applications	Unit loads MPa	Applications	Unit loads MPa
Electric motors	0.8 – 1.5	Air compressors Main bearing	1.0 - 2.0
Steam turbines	1.0 – 2.0	Air compressors Crank pin bearing	2.0 – 4.0
Gear reducers	0.8 – 1.5	Centrifugal pumps	**0.6 – 1.2**

Table 1 Unit loads for journal bearings

- Recommended l/d ratio for centrifugal pumps is 0.75 to 2.
- A value of l/d = 0.75 is chosen.

L = 0.75

d = 0.75x120 = 80mm

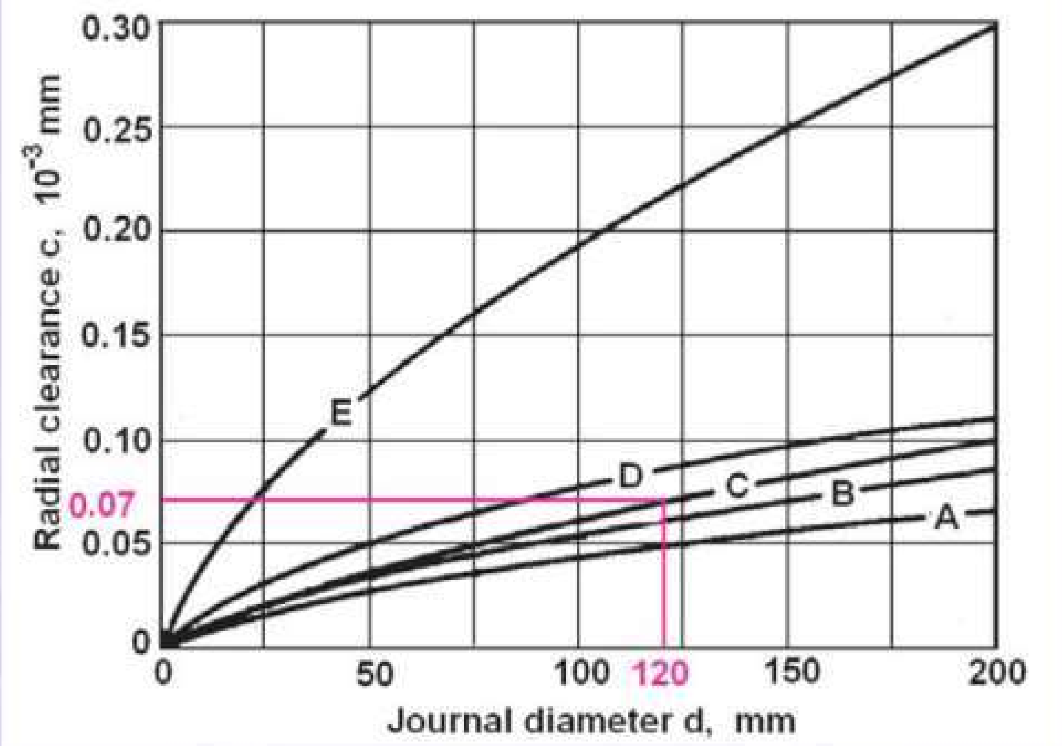
p = F/ l d

= 8000 / 80 x 120

= 0.833 MPa

which is within the range for centrifugal pump 0.6 to 1.2 MPa

Choosing cast bronze material for the bearing, the recommended clearance is coming under C curve of the figure …



Recommended radial clearance for cast bronze bearings

- Electric motors, generators, and similar types of machinery using ground journals in broached or reamed cast-bronze bearings (0.4 to 0.8 μm rms finish)
- From the figure, the recommended clearance for 120 mm diameter journal is 0.07 mm.

$h_o \geq 0.005 + 0.00004\ d$

$= 0.005 + 0.00004 \times 120$

$= 0.0098 mm$

The peak to valley height of roughness

$R_1 = 1.5\ \mu m$ for fine ground journal and

$R_2 = 2.5\ \mu m$ lapped bearing assumed.

$h_o > 0.5\ (R_1 + R_2) = 0.5\ (1.5+2.5) = 2\ \mu m$

Hence ,

$h_o = 0.012$ is aimed at which is at least 6 times the average peak to valley roughness of journal and bearing and safe working regime for hydro-dynamic lubrication.

The recommended viscosity of oil for the centrifugal pump application is 30 – 80 cP.

Hence from the chart SAE 30 oil is chosen.

Assuming the bearing to operate between 50 to 60 OC and average oil temperature of 55 OC,

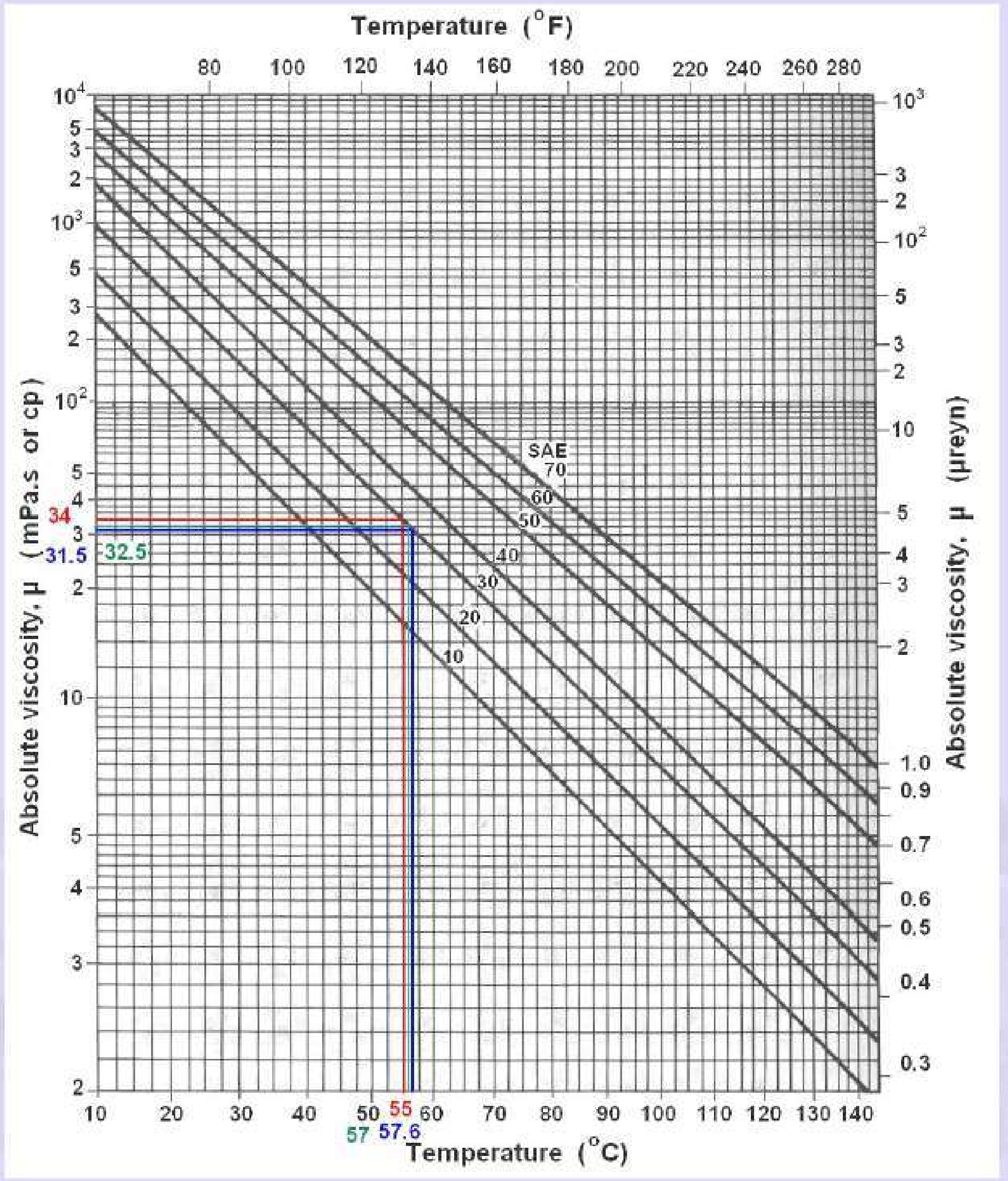
μ = 34 cP from the figure

Clearance ratio of ψ

for p < 8 MPa and v > 3 m /s.

(c/r) =2x10 -3 assumed.

Or r/c = 500.



Viscosity – temperature curves of SAE graded oils

Clearance ratio of ψ for p < 8 MPa and v > 3 m /s.

(c/r) = 2x10 -3 assumed.

Or r/c = 500.

$$S = \left(\frac{r}{c}\right)^2 \left(\frac{\mu n}{p}\right) = (500)^2 \left(\frac{34 x 10^{-3} x 29}{0.833 x 10^6}\right) = 0.296$$

Table 4.2a Clearance ratio: $\psi = c/r$ in 10^{-3}

Working pressure p MPa	Peripheral speed m/s		
	Low < 2	Medium – 2 to 3	High >3
Low to medium p< 8 MPa	0.7-1.2	1.24 – 2.0	2 - 3
High p>8 MPa	0.3 – 0.6	0.8 – 1.4	1.5 – 2.5

S = 0.296 and l/d = 0.75,

$T_{var} = \gamma\, C_H\, (\Delta T/p)$ =26.5 from Fig.

$\Delta T = 26.5\ p/\ \gamma\ C_H$

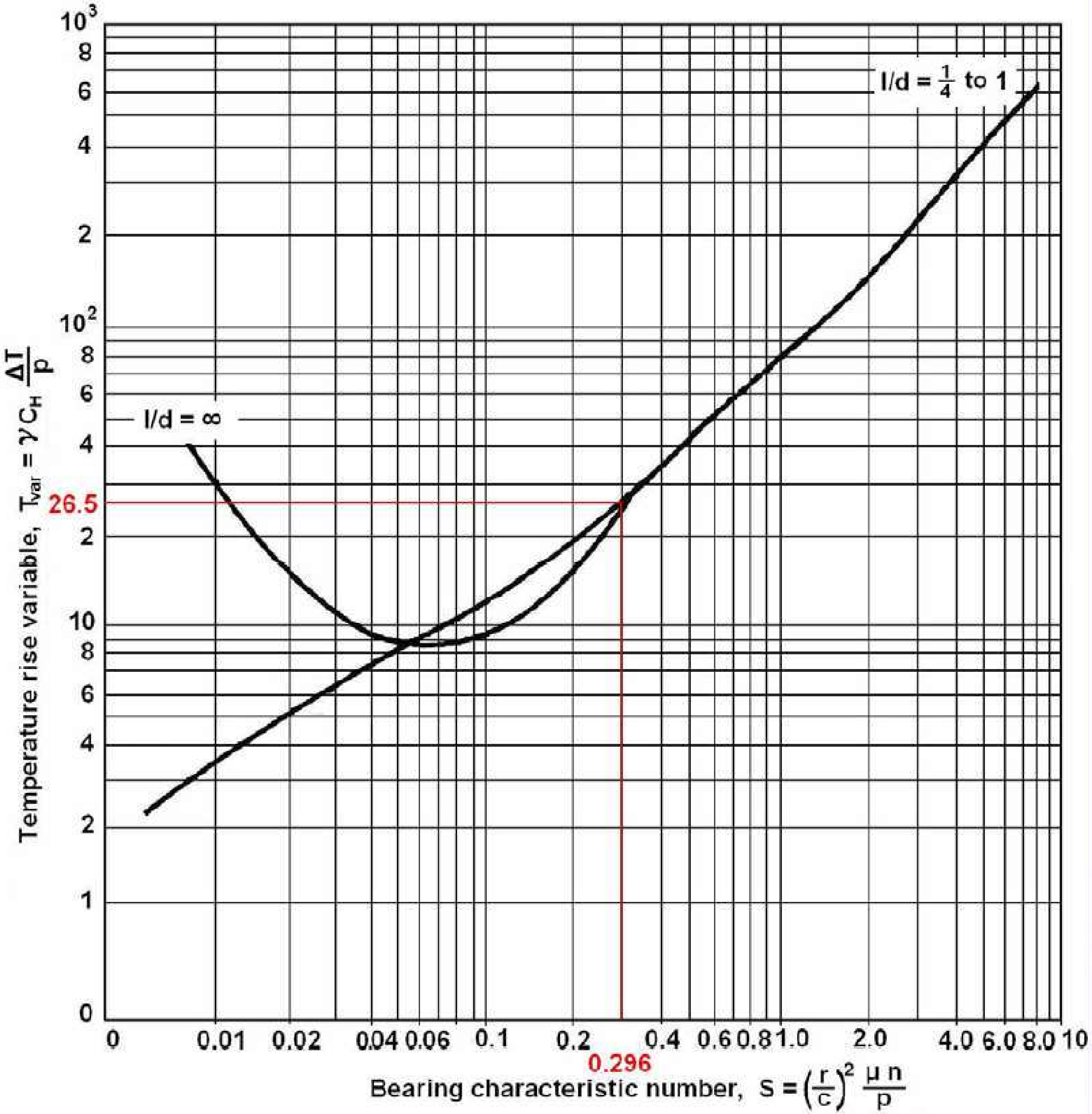
$= 26.5 \times 0.833 \times 10^6 / 861 \times 1760$

$= 14.6^oC$

$T_{av} = T_i + 0.5\ \Delta T$

$= 50 + 0.5 \times 14.6$

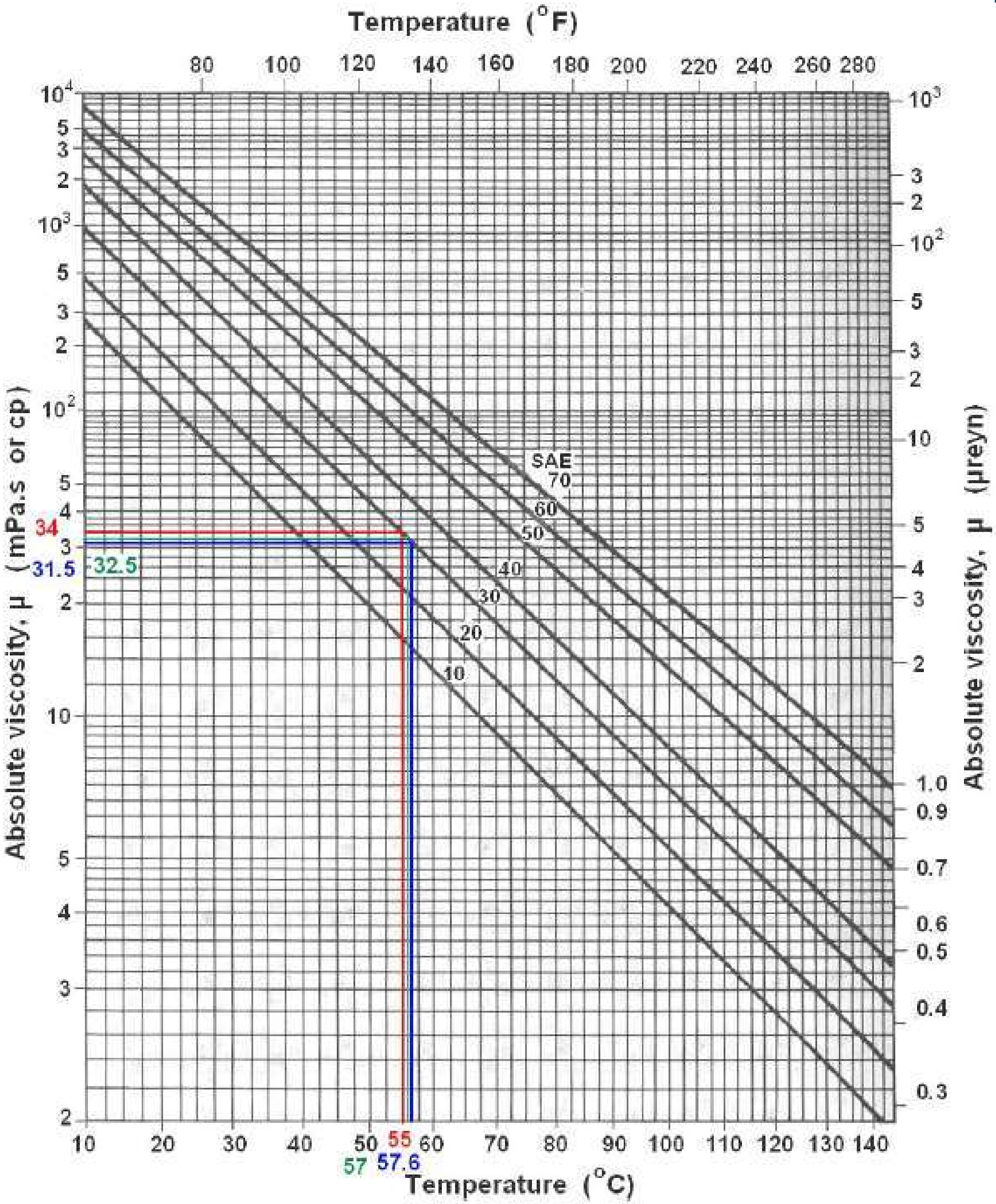
$= 57.3^oC$



For T_{av} = 57.3°C,

μ = 31.5cP from Fig.

Recalculated S = 0.274



For S = 0.274 and l/d = 0.75,

T_{var} = 24 from Fig.

$\Delta T = 24\ p/\ \gamma\ C_H$

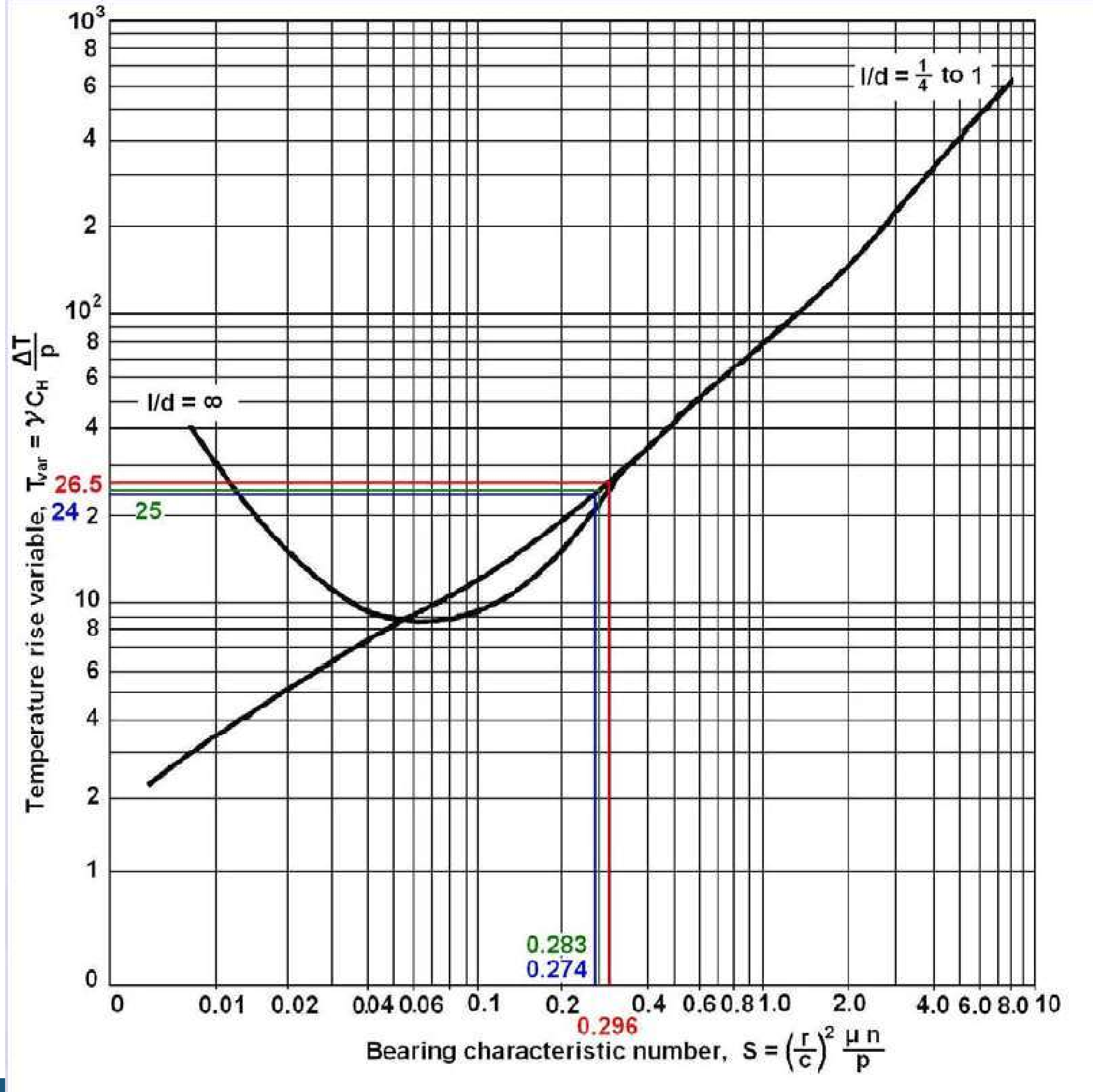
$= 24 \times 0.833 \times 10^6 / 861 \times 1760$

$= 13.2^oC$

$T_{av} = T_i + 0.5\ \Delta T$

$= 50 + 0.5 \times 13.2$

$= 56.6^oC$



- For T_{av} = 56.6°C, μ = 32cP, S =0 283, T_{var} = 24, ΔT =13.8°C

  $T_{av} = T_i + 0.5\ \Delta T = 50 + 0.5 \times 13.8 = 56.9^{o}C$

For T_{av} = 56.9°C, μ = 32.5cP, S =0. 283, h_o/c = 0.492; T_{var}=25;

Q / r c n l = 4.45; Q/Q_{max} = 0.605; (r/c) f = 6.6;

$P/_{pmax}$ = 0.42; Φ = 54.8°; θpo =78°; θ_{pmax} = 17.8°;

$h_o = 0.492 \times c = 0.492 \times 0.12 = 0.059$ mm

$f = 6.6(c/r) = 6.6\times 2.0 \times 10^{-3} = 0.0132$

$\Delta T = 25\ p/\ \gamma\ C_H = 24 \times 0.833 \times 106 / 861\times1760 = 13.74^{o}C$

$T_{av} = T_i + 0.5\ \Delta T = 50 + 0.5x\ 13.74 = 56.87^{o}C = 56.9^{o}C$

$Q = 4.45 \text{ x rcnl} = 4.48 \text{ x } .06 \text{ x}0.00012\text{x}29\text{x}0.08$

$= 7.43 \text{ x } 10^{-5}\ m^3/s = 73.4\ cm^3/s$

. $Q_s = 0.605 \text{ x } 73.4 = 45\ cm^3/s$

$p_{max} = p/0.42 = 0.833/0.42 = 1.98$ MPa

Bearing diameter: 120 H7 - 120.00 / 120.035

Journal diameter-120 f8 -119.964 / 119.910

Fit = 120 H7/f8

Frictional power loss: f.Fv = 0.0132x8000x10.93=1154 W

. Final details of the designed bearing are given in tabular form

d=120mm	l = 80mm	l/d = 0.75	SAE 30 oil	C= 120μm
h_o =59 μm	p=0.833MPa	p_{max}=1.98MPa	T_{av}=56.9°C	T_i = 50°C
φ = 54.8°	**θ_{pmax} = 17.8°**	**θ_{po} =78°**	Q =73.4cc/s	Q_s=45 cc/s
Bearing material	Cast Bronze Reamed and honed	f = 0.0132 Fit 120 H7/ f8	Journal Hardened & ground	T_H =63.8°C μ = 32.5 cP

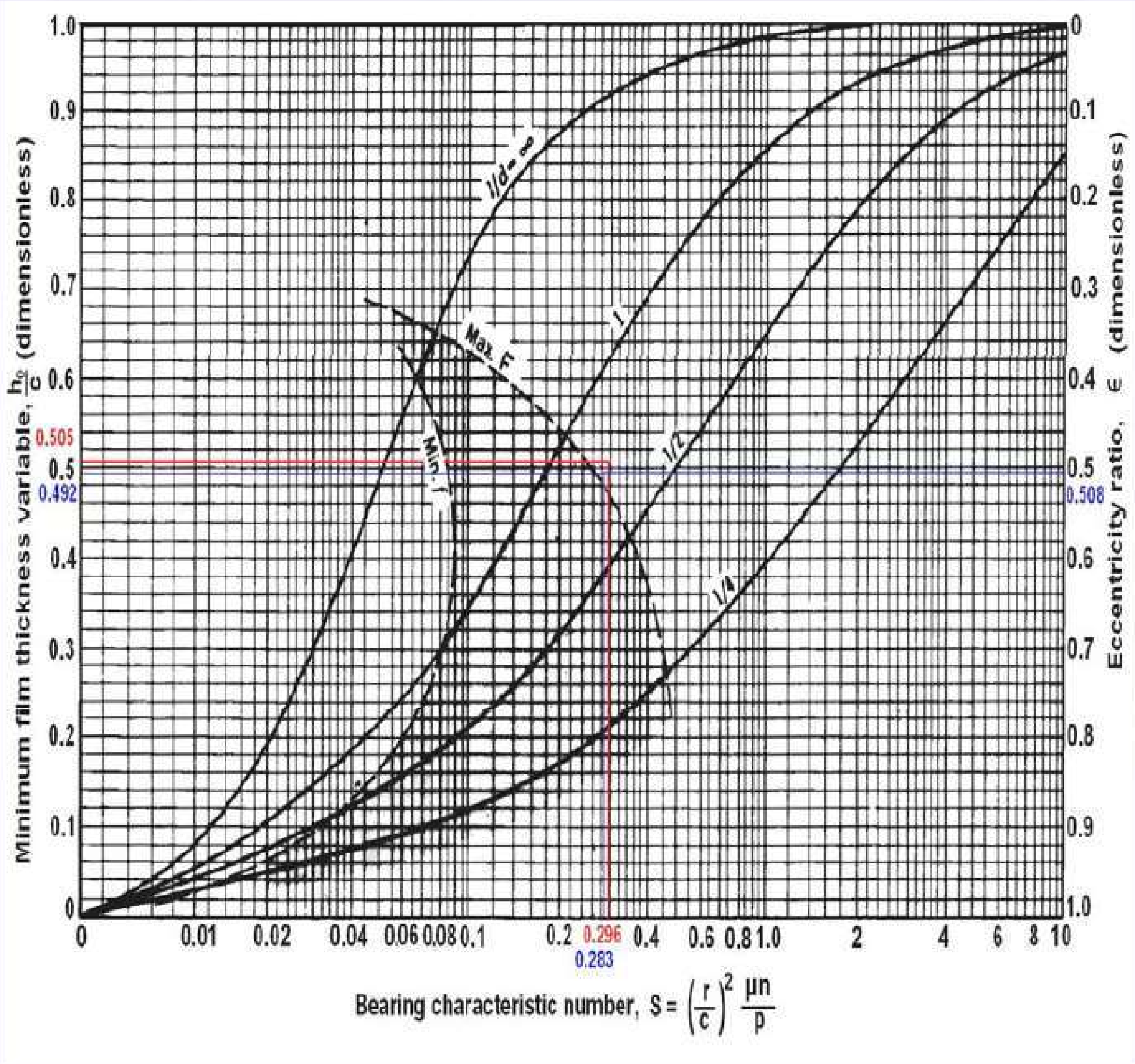


Fig.2.8b Chart for minimum film thickness variable and eccentricity ratio. The left shaded zone defines the optimum h_o for minimum friction; the right boundary is the optimum h_o for maximum load

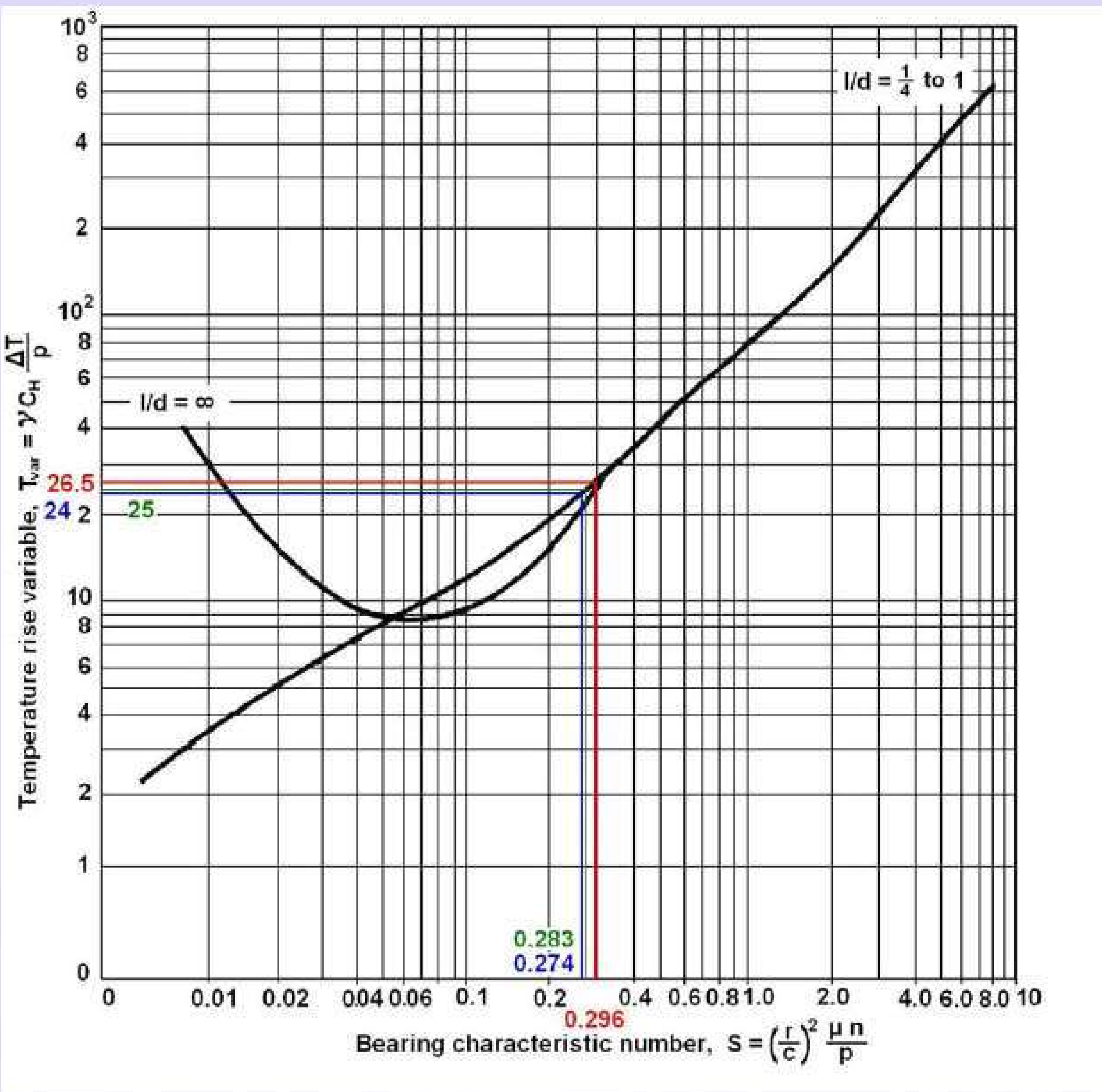


Fig. 2.20d Chart for temperature variable, $T_{var} = \gamma C_H (\Delta T/p)$

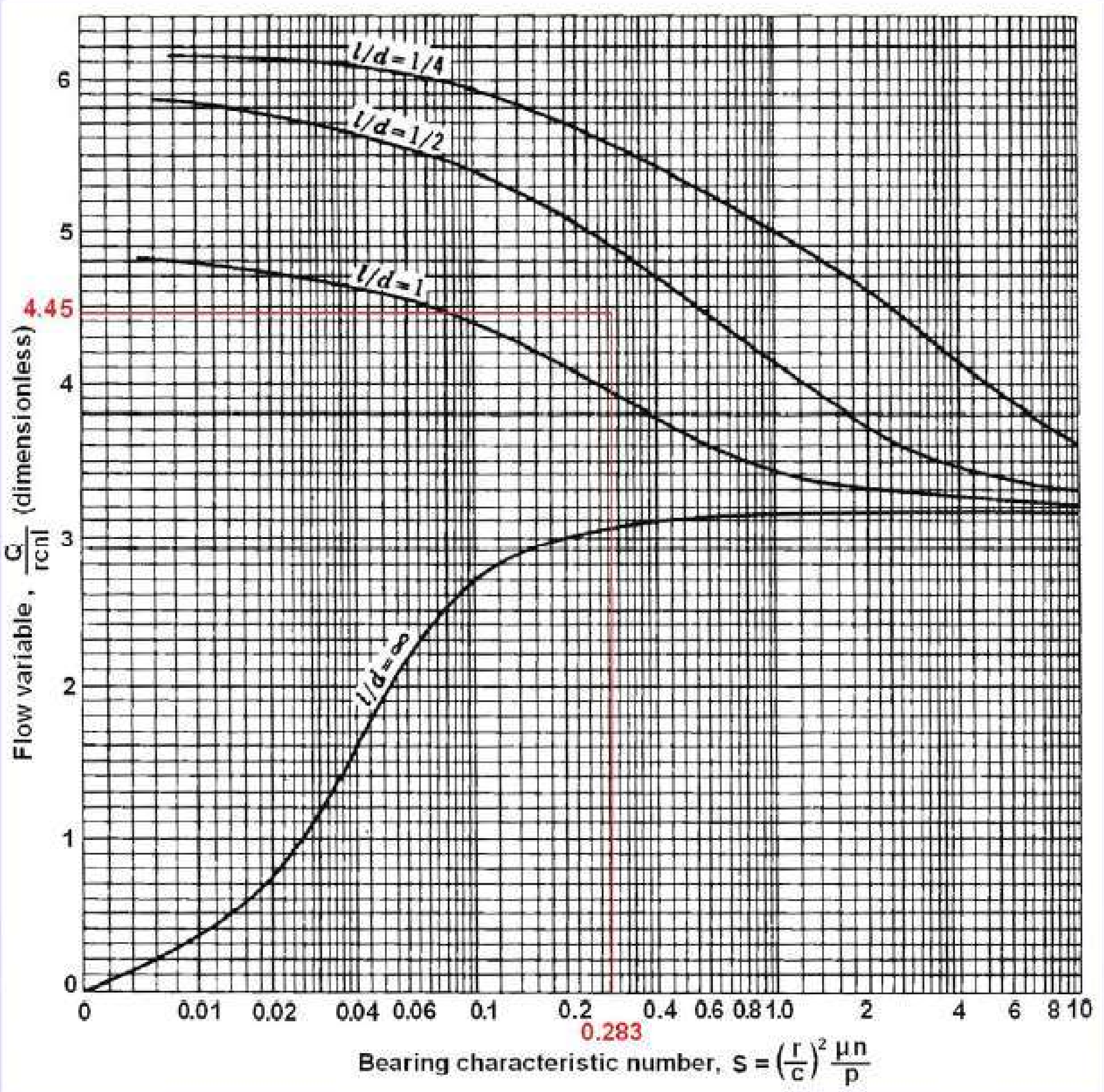


Fig. 2.12b Chart for flow variable.

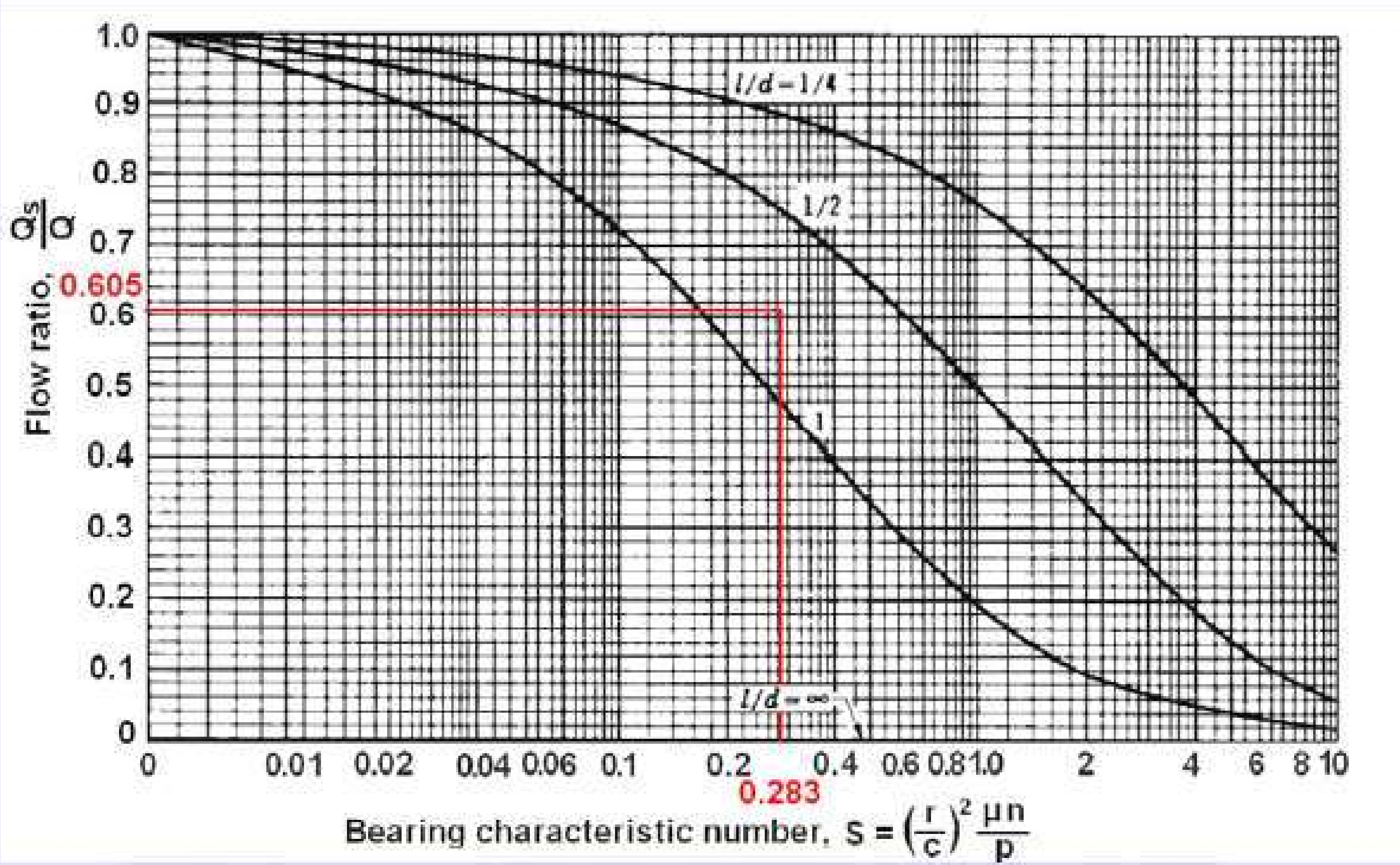


Fig.2.13b Chart for determining the ratio of side flow to total flow

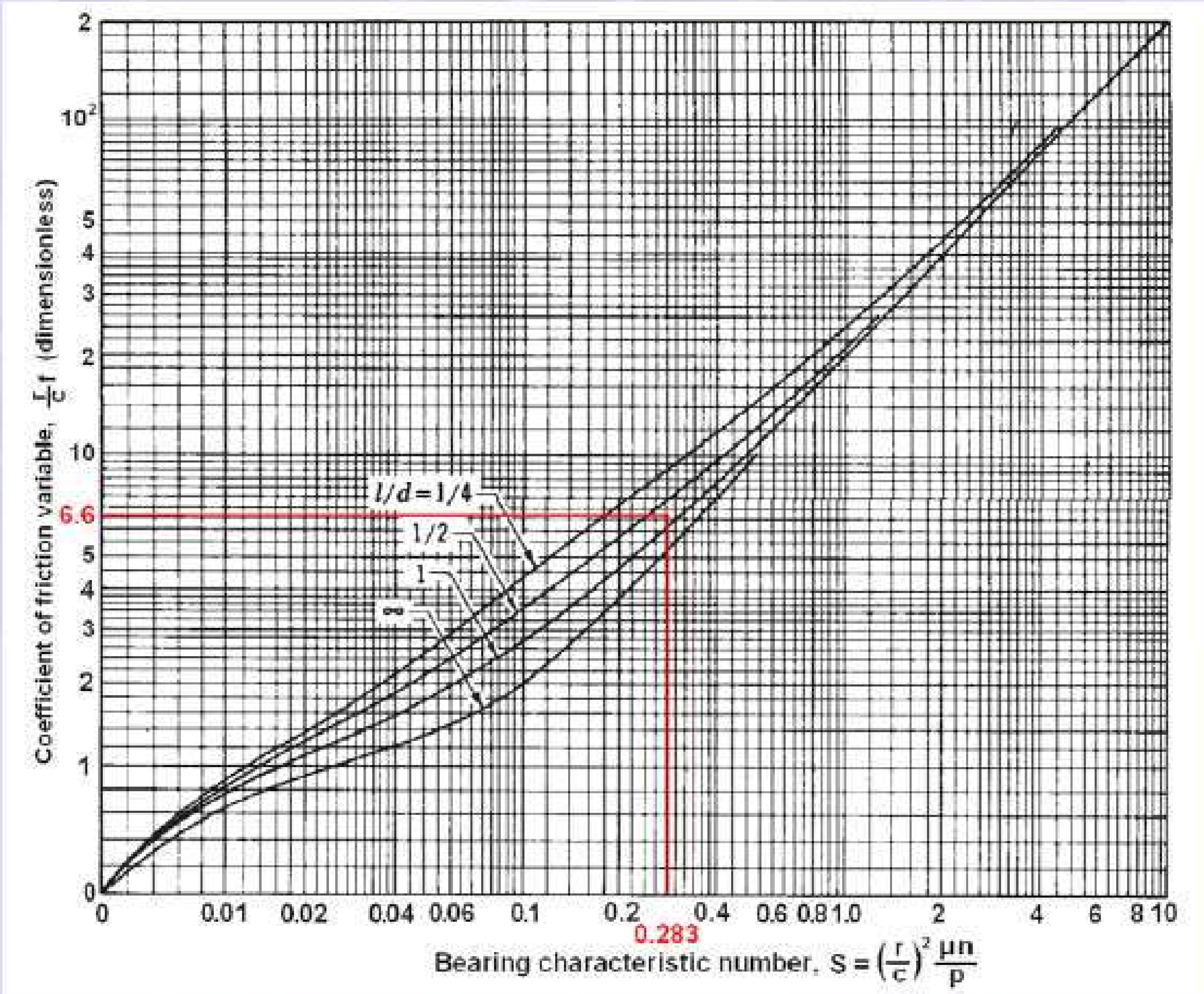


Fig. 2.11b Chart for coefficient of friction variable

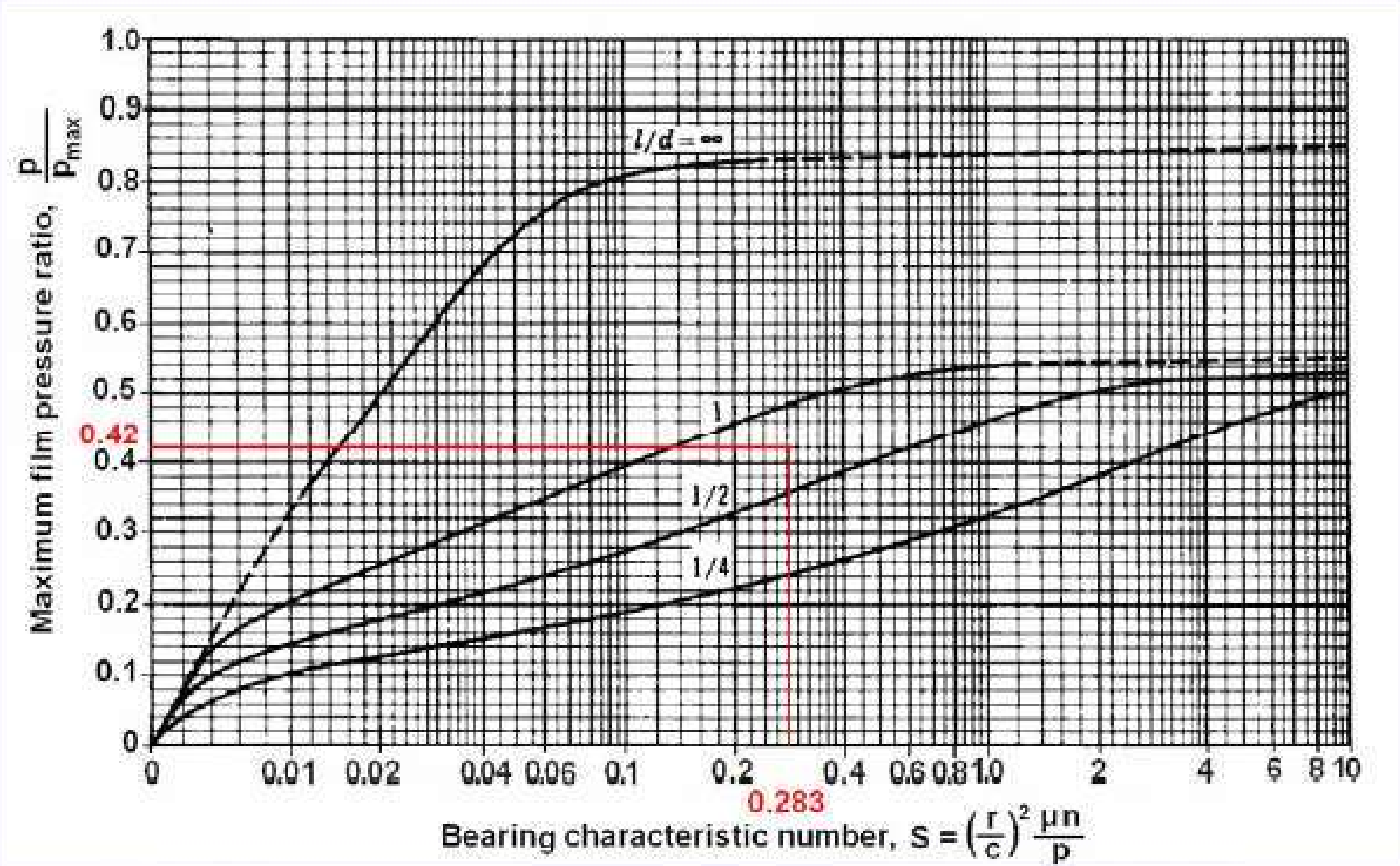


Fig. 2.14a Chart for determining the maximum film pressure

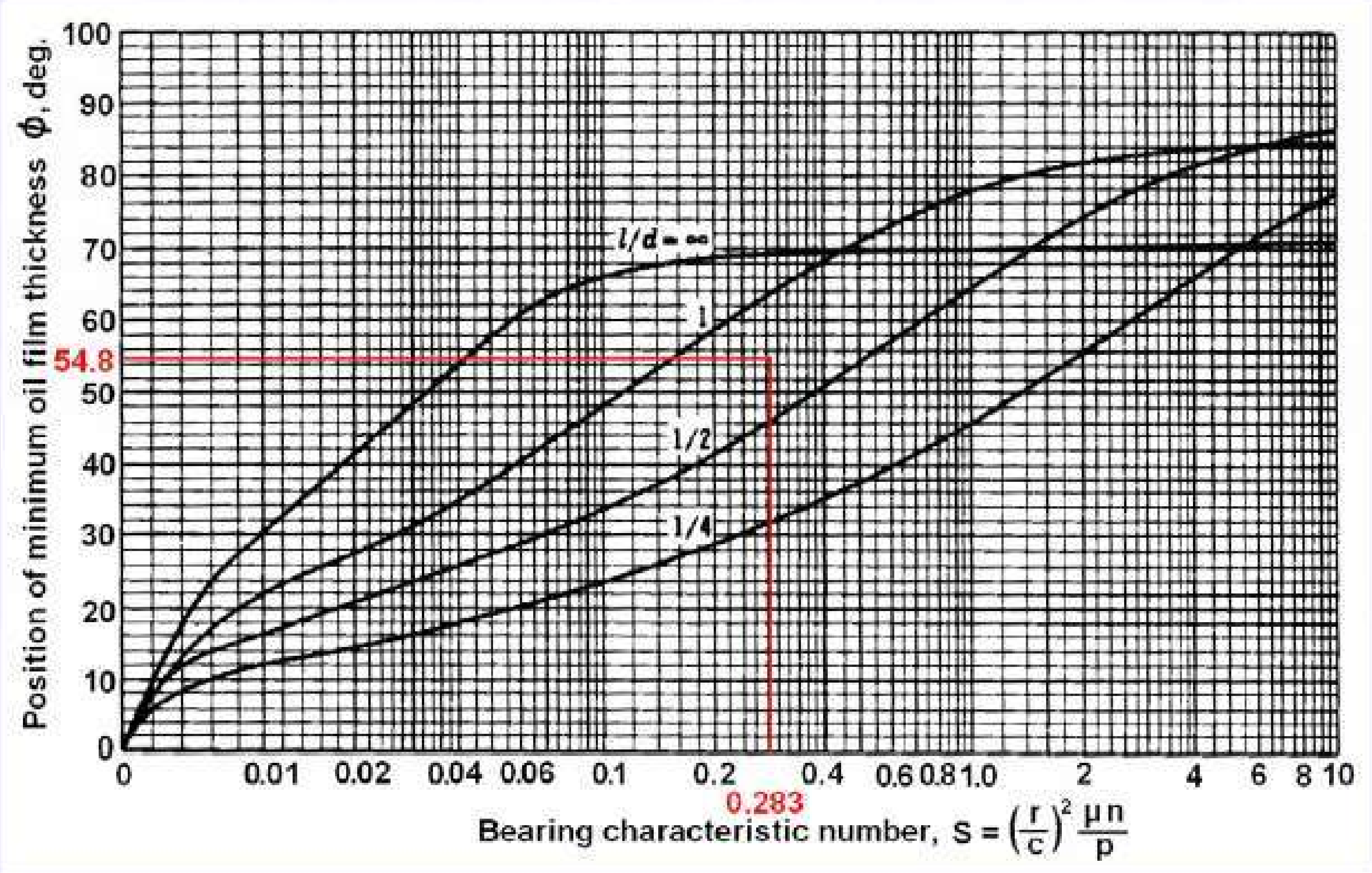


Fig.2.9b Chart for determining the position of minimum film thickness h_o

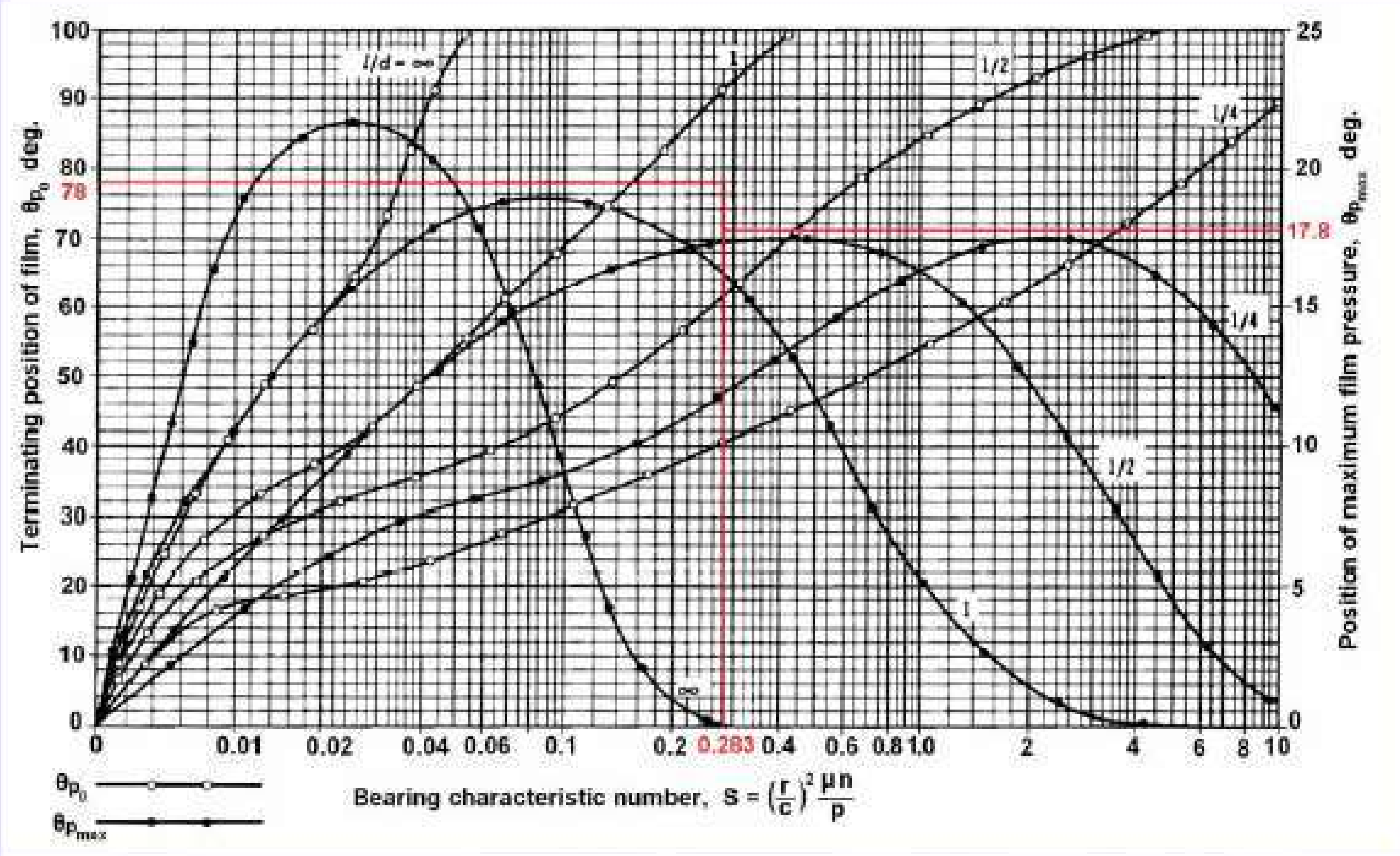
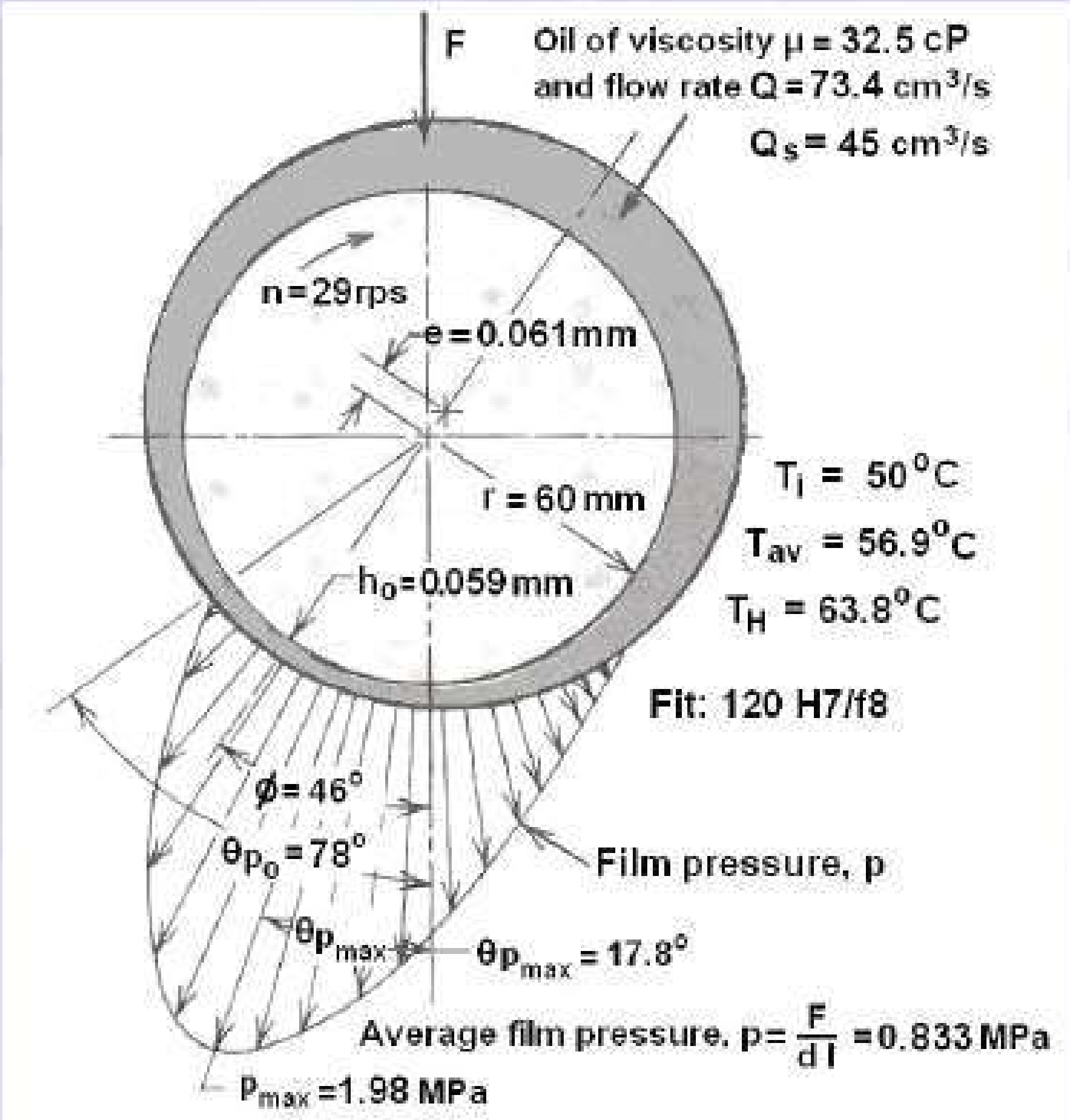


Fig. 2.15b Chart for finding the terminating position of oil film and position of maximum film pressure



ig 4.4 Journal position under stable hydrodynamic lubrication condition problem1

----- End of problem 1---

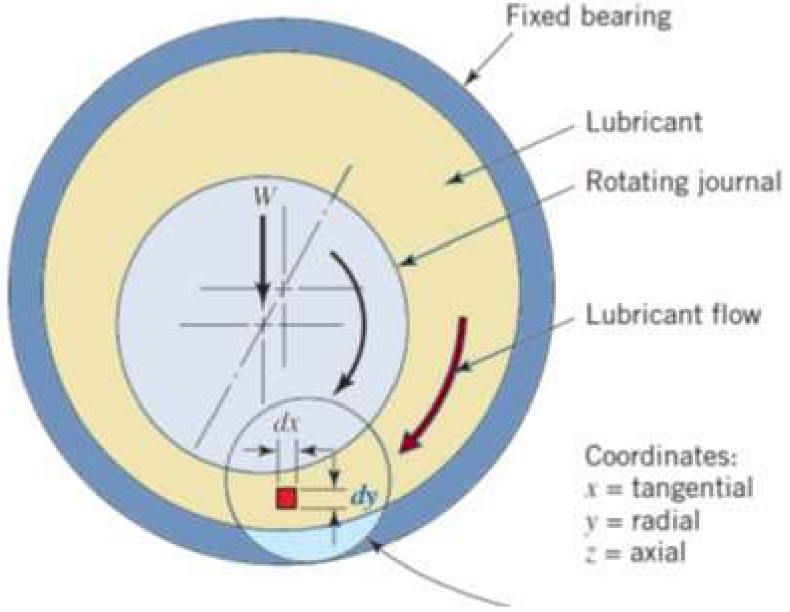
Lecture - 10

Hydrodynamic Lubrication

By

Prof. M. Naushad Alam

MACHINE DESIGN II
MEC 3110



MECHANICAL ENGINEERING DEPT.
A.M.U. ALIGARH

BEARING MATERIALS

- Bearing materials constitute an import part of any journal bearing.
- Their significance is at the start of the hydro- dynamic lubrication
  - when metal to metal contact occurs
  - or during mixed and boundary lubrication period.

DESIRABLE PROPERTIES OF A GOOD BEARING MATERIAL

1. Conformability (low elastic modulus) and deformability (plastic flow) to relieve local high pressures caused by misalignment and shaft deflection.

2. Embeddability or indentation softness, to permit small foreign particles to become safely embedded in the material, thus protecting the journal against wear.

3. Low shear strength for easy smoothing of surface asperities.

4. Adequate compressive strength and fatigue strength for supporting the load and for enduring the cyclic loading as with engine bearings under all operating conditions.

5. Should have good thermal conductivity to dissipate the frictional heat and coefficient of thermal expansion similar to the journal and housing material.

6. It should be compatible with journal material to resist scoring, welding and seizing.

7. Should have good corrosion resistance against the lubricant and engine combustion products.

COMPOSITION OF BEARING MATERIALS

- Babbits are the most commonly used bearing materials. Babbitts have excellent conformability and embeddability, but have relatively low compressive and fatigue strength, particularly above 77°C.
- Babbitts can seldom be used above about 121°C.
- Other materials such as tin bronze, leaded bronze, copper lead alloy, aluminium
- bronze, aluminium alloys and cast iron are also used in many applications.

Bearing Material Compositions

Widely used bearing material compositions are given below:

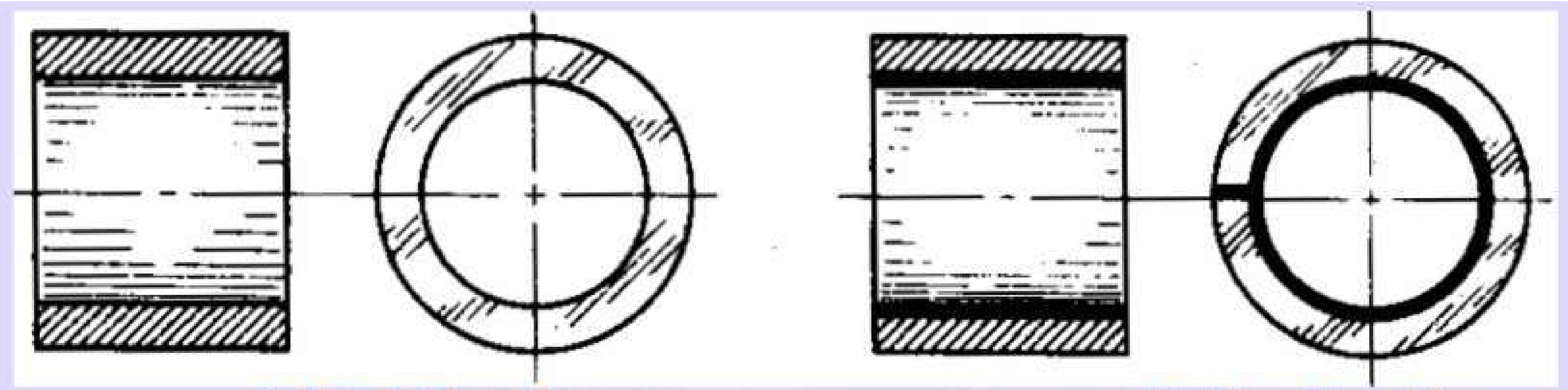
a. Tin-base babbitts with 89% Sn, 8% Pb and 3% Cu,

b. Lead- base babbitts with 75% Pb, 15% Sb and 10% Sn,

c. Copper alloys such as Cu- 10% to 15% Pb.

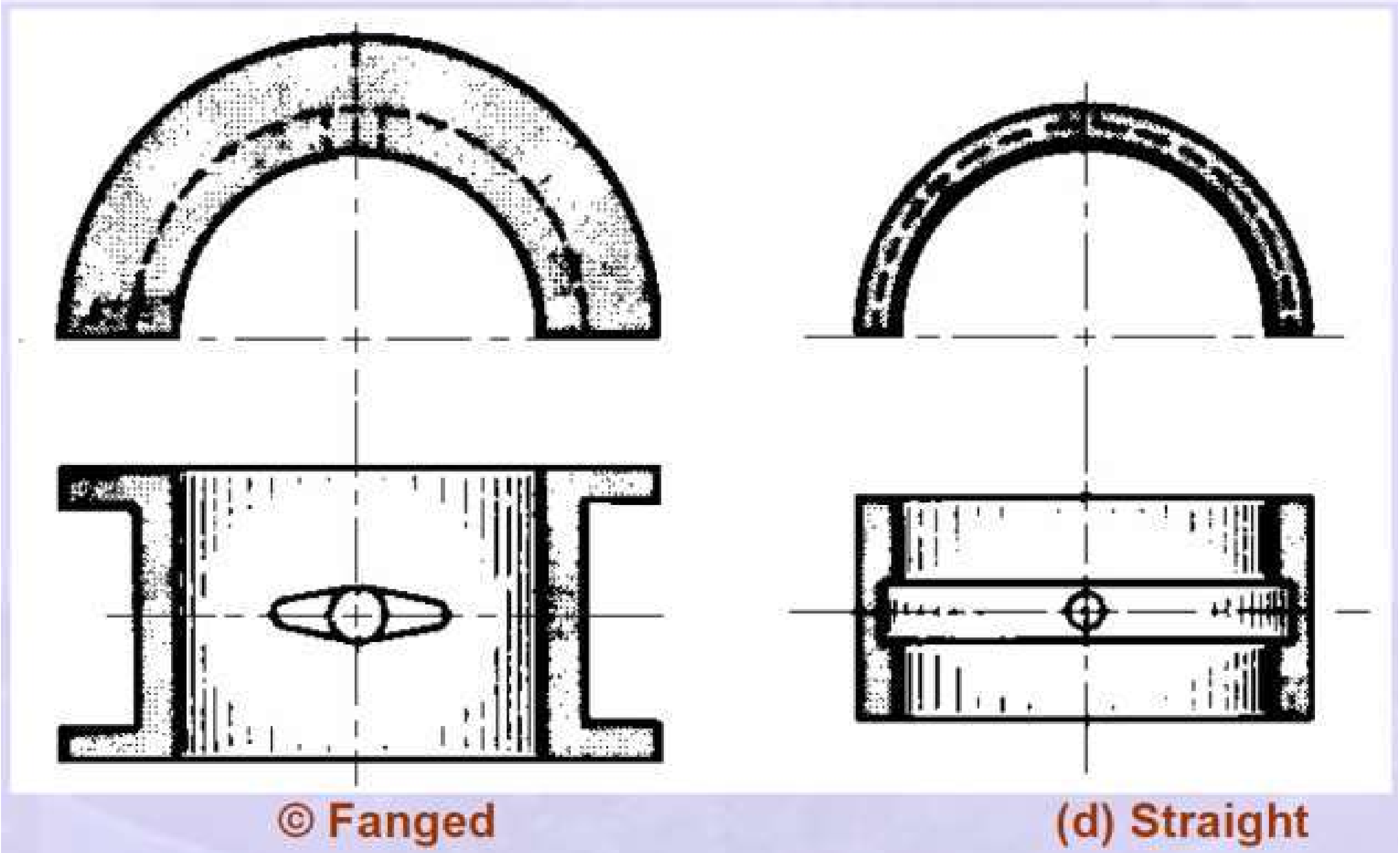
Bimetal and trimetal bearings

- Bimetal and trimetal bearings are used in engine application in order to:
  - reduce the size of the bearing
  - obtain good compatibility
  - achieve more load capacity.
- The bearings can be of solid bushings or lined bushings.



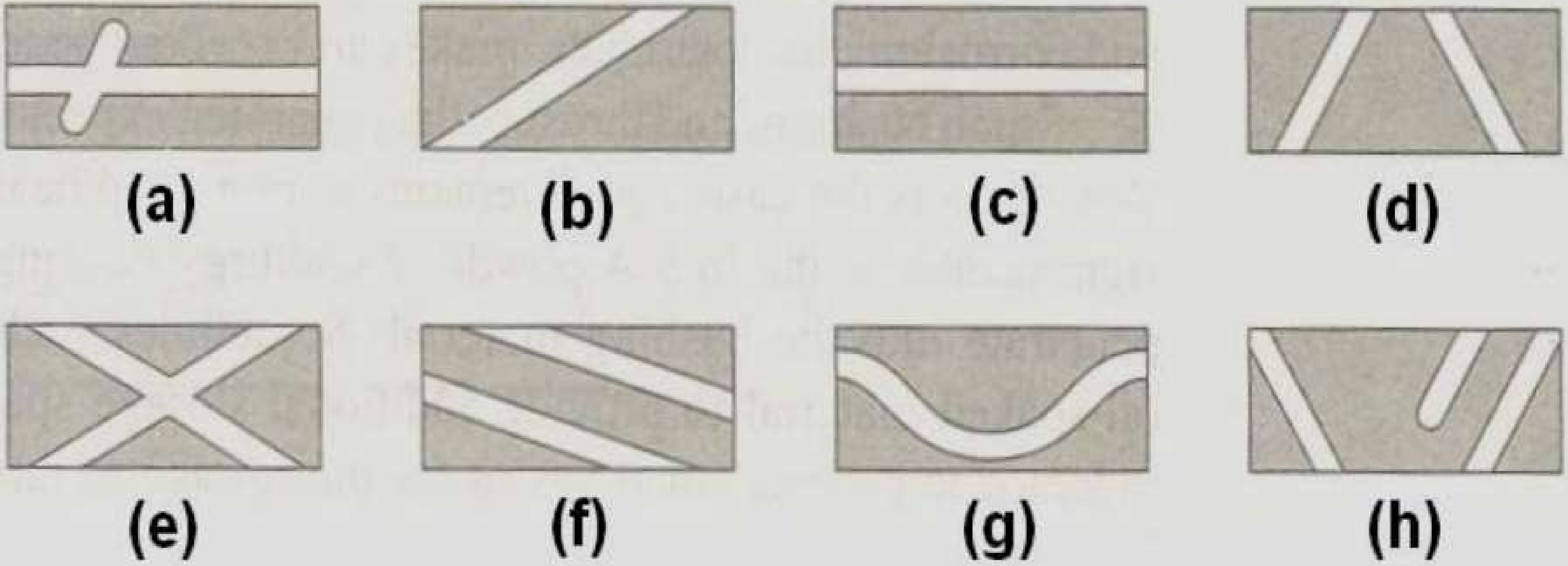
(a) Solid bushing (b) Lined bushing

- Bearings are often grooved to facilitate the supply of lubricant to the surface of the journal



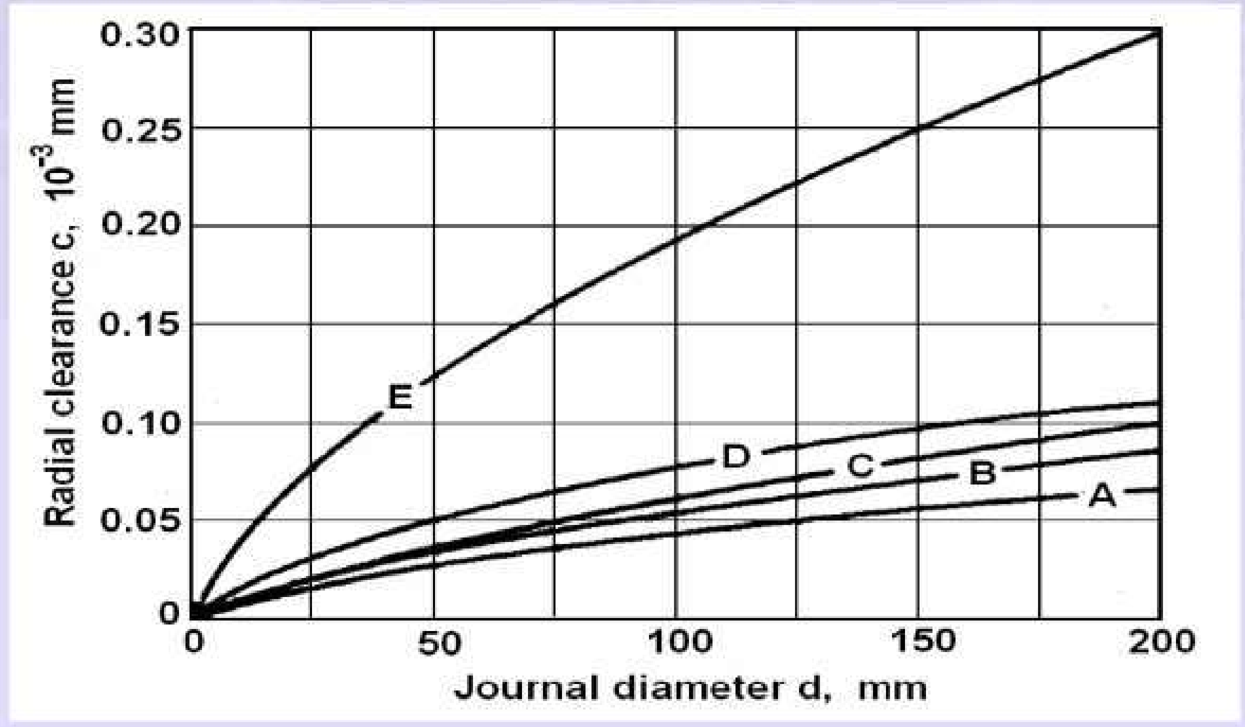
Various types of bush bearings

Various groove pattern used in industry



Developed views of typical groove patterns

- Recommended radial clearances for cast bronze bearings are shown in Figure.

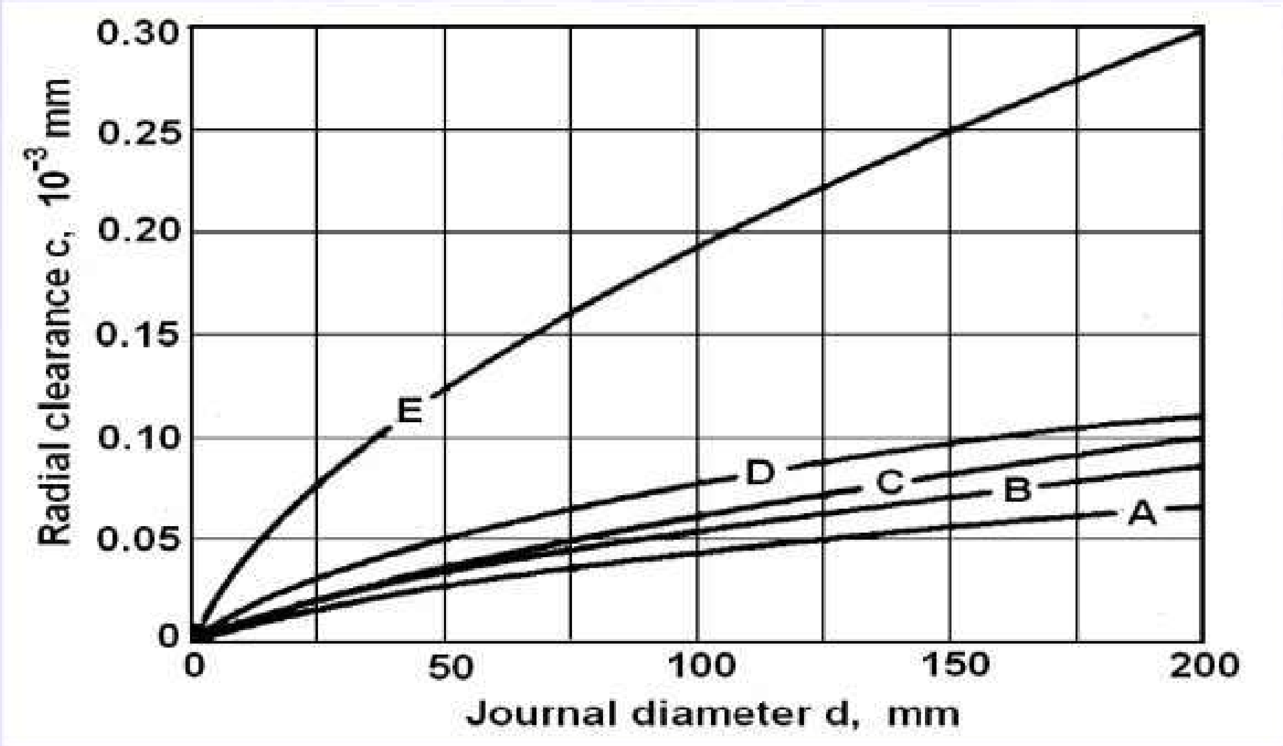


Recommended radial clearance for cast bronze bearings

- A – Precision spindles made of hardened ground steel, running on lapped cast bronze bearings (0.2 to 0.8 µm rms finish) with a surface velocity less than 3 m/s.

B - Precision spindles made of hardened ground steel, running on lapped cast bronze bearings (0.2 to 0.4 μm rms finish) with a surface velocity more than 3 m/s.

C- Electric motors, generators, and similar types of machinery using ground journals in broached or reamed cast-bronze bearings (0.4 to 0.8 μm rms finish)



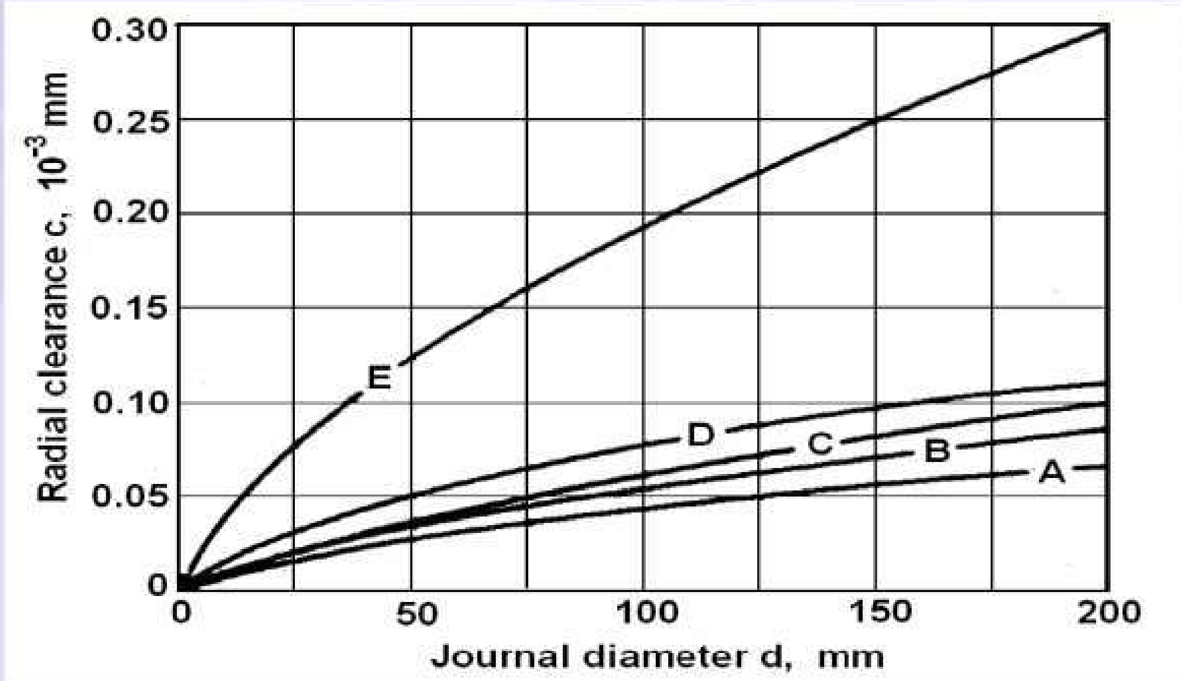
•

Recommended radial clearance for cast bronze bearings

•••

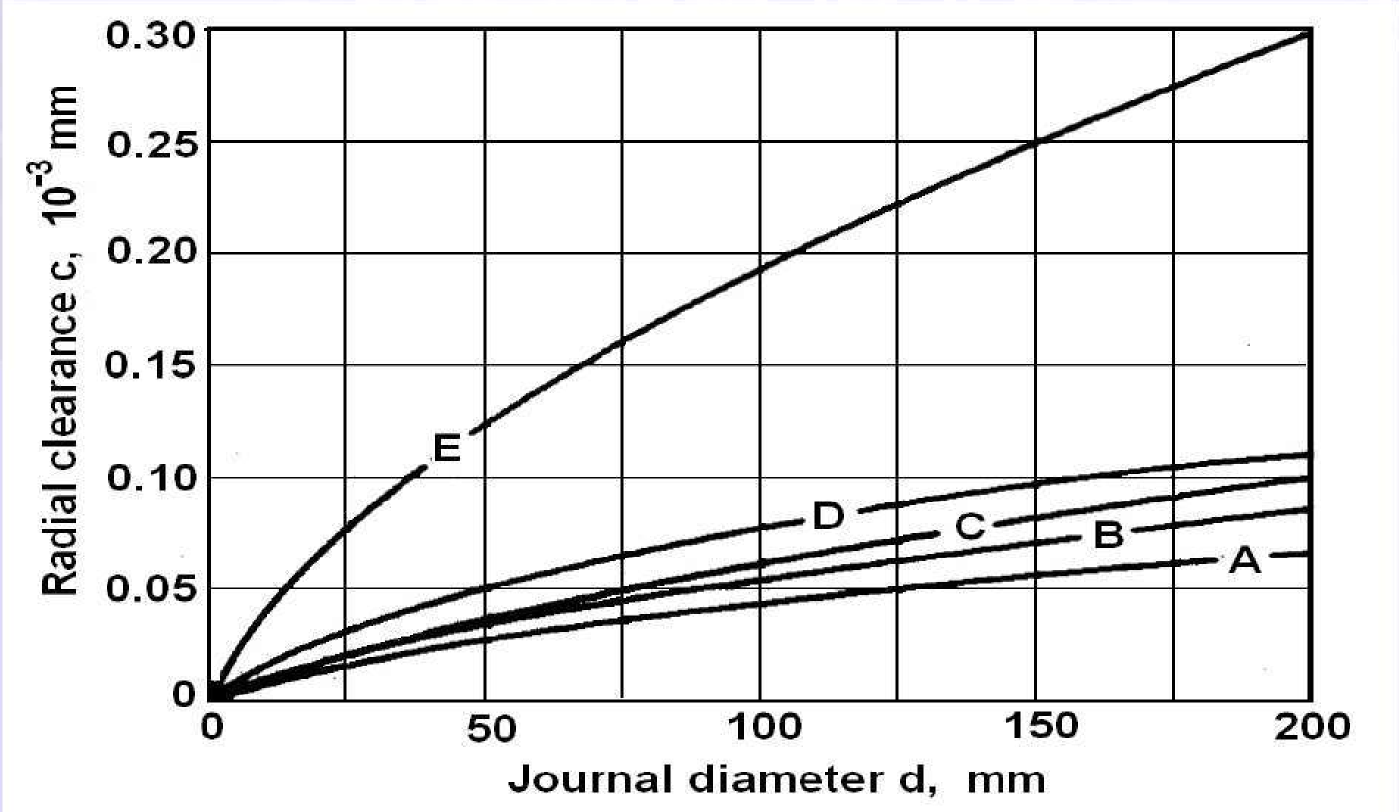
D – General machinery which continuously rotates or reciprocates and uses turned or cold rolled steel journals in bored and reamed cast-bronze bearings (0.8 to 1.6 μm rms finish)

E- Rough service machinery having turned or cold rolled steel journals in bored and reamed cast-bronze bearings (0.8 to 1.6 μm rms finish)



Recommended radial clearance for cast bronze bearings

•



Recommended radial clearance for cast bronze bearings

Table 4.5(a) Bearing material properties

Material	Maximum pressure p_{max} MPa	Maximum Temperature T_{Bmax} °C	Maximum Speed V_{max} m/s	Maximum pv value MPa.m/s
Cast Bronze	31	165	7.5	1.75
Sintered bronze	31	65	7.5	1.75
Sintered Fe	55	65	4	1.75
Pb-bronze	24	150	7.6	2.1
Sintered Fe-Cu	28	65	1.1	1.2

Table 4.5(b) Bearing material properties

Material	Maximum pressure p_{max} MPa	Maximum Temperature T_{Bmax} °C	Maximum Speed V_{max} m/s	Maximum pv value MPa.m/s
Cast iron	4	150	1.5	0. 5
Hardenable Fe-Cu	55	--	0.2	2.6
Bronze-iron	17	--	4.1	1.2
Lead- iron	7	--	4.1	1.8
Aluminium	14	--	6.1	1.8

Table 4.5(c). Bearing material properties

Material	Maximum pressure p_{max} MPa	Maximum Temperature T_{Bmax} °C	Maximum Speed V_{max} m/s	Maximum pv value MPa.m/s
Phenolics	41	93	13	0.53
Nylon	14	93	3	0.11
TFE	3.5	260	0.25	0.035
Filled TFE	17	260	5.1	0.35
TFE fabric	414	260	0.76	0.88

Table 4.5(d) Bearing material properties

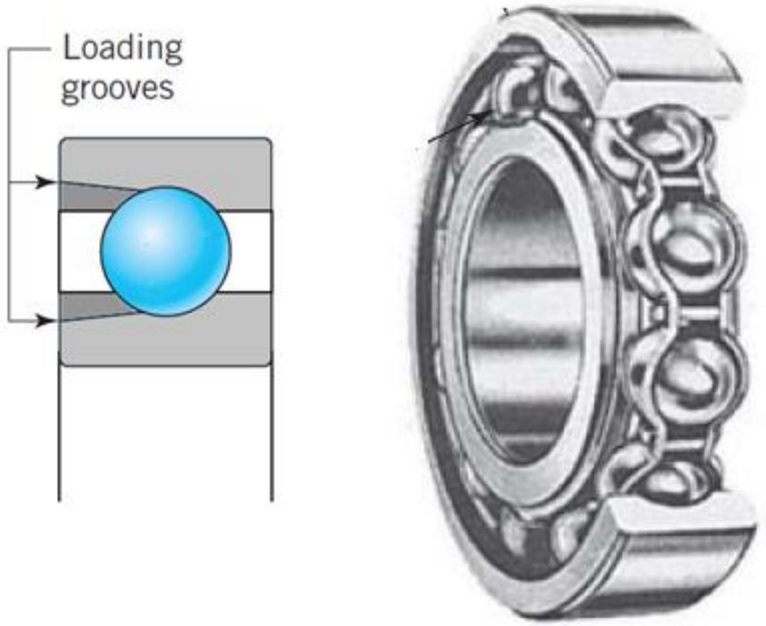
Material	Maximum pressure p_{max} MPa	Maximum Temperature T_{Bmax} °C	Maximum Speed V_{max} m/s	Maximum pv value MPa.m/s
Polycarbonate	7	104	5.1	011
Acetal	14	93	3	0.11
Carbon graphite	4	400	13	0.53
Rubber	0.35	66	20	-------
Wood	14	71	10	0.42

MACHINE DESIGN II

MEC 3110

LECTURE - 11

ROLLING-ELEMENT BEARINGS

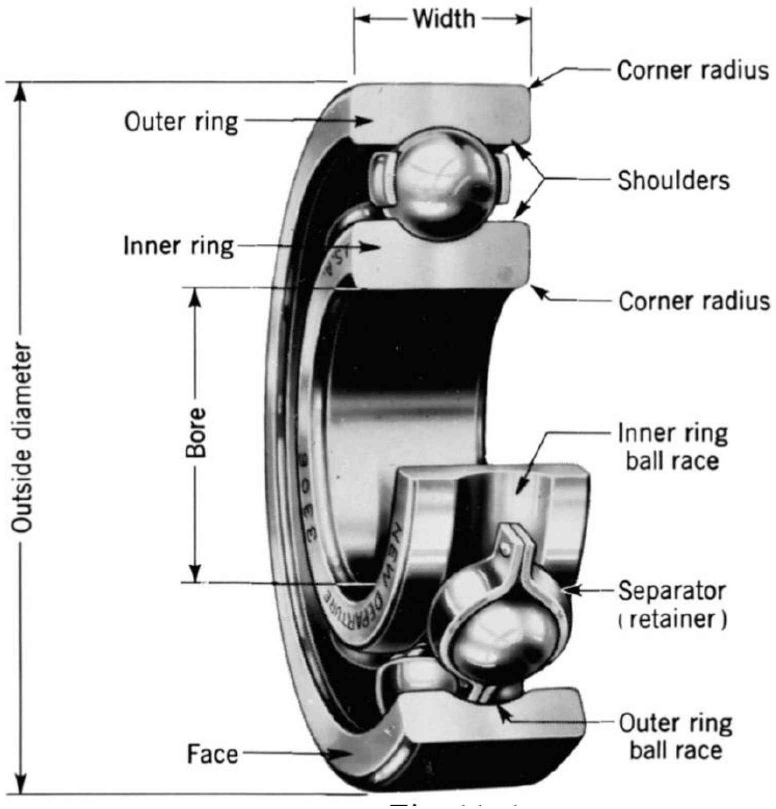


PROF. M. NAUSHAD ALAM

MECHANICAL ENGINEERING DEPARTMENT
A.M.U. ALIGARH

ROLLING-ELEMENT BEARINGS

- Rolling-element bearings are either ***ball bearings*** or ***roller bearings***.
- *These* are capable of running at higher speeds, and can carry greater loads.
- The shaft and outer members of the bearing are separated by balls or rollers.
- Thus, **rolling friction** is substituted for **sliding friction.**
- Rolling-element bearings are also known as "**antifriction bearings**". because these bearings provide lower friction than fluid-film bearings.
- With normal operating loads,
  rolling-element bearings (without seals)
  typically provide coefficients of friction
  between 0.001 and 0.002.



…

- Since the bearing contact surface areas are small, and therefore the contact stresses are high.
- The loaded parts of rolling-element bearings therefore have to be made of hard and high-strength materials, as compared to the outer member.
- These parts include inner and outer rings and the balls or rollers.
- An additional component of the bearing is usually a retainer or separator.
- The retainer keeps the balls or rollers evenly spaced and separated.

...

Advantages:

- Ideally suited for applications with **high starting loads**.

  For example,

  the use of roller bearings to support rail car axles,
  eliminates the need for an extra locomotive,
  to get a long train started from rest.

- Another advantage is that they can be "**preloaded**".

  Mating bearing elements are pressed together
  rather than operating with a small clearance.
  This is important when precise positioning of the
  rotating member is required.

CLASSIFICATION
Rolling-element Bearings

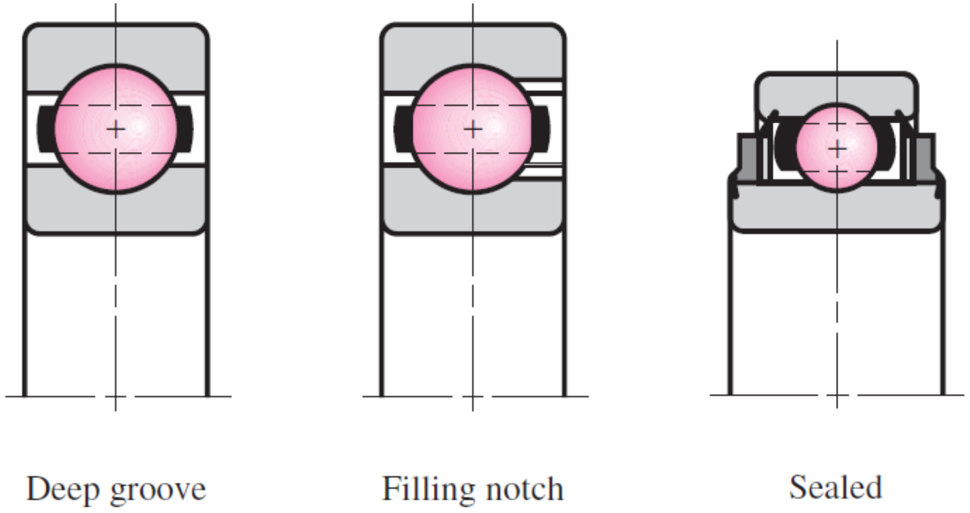
- Rolling-element bearings can be classed according to the **type of loading** into the following three categories:
  1. *Radial* bearings.
  2. *Thrust* or axial bearings.
  3. *Angular-contact* bearings.

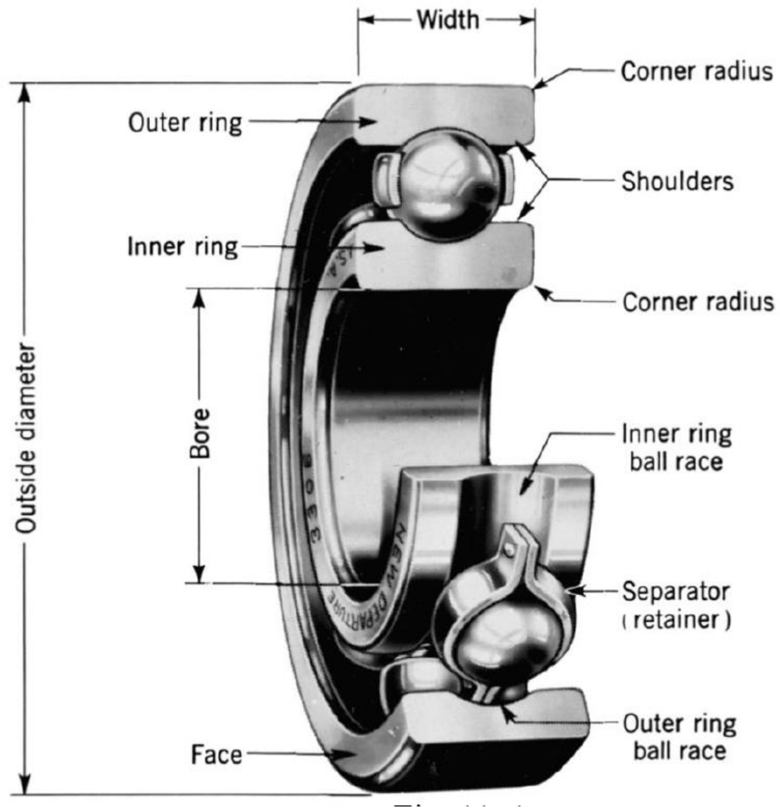
- Roller bearings can also be classified by the **roller configuration** into the four types :
  1. *Cylindrical* roller bearings,
  2. *Spherical* roller bearings,
  3. *Tapered* roller bearings,
  4. Needle *bearings*.

TYPES OF ROLLING-ELEMENT BEARINGS

1. *Radial* bearings

for carrying loads that are primarily radial.



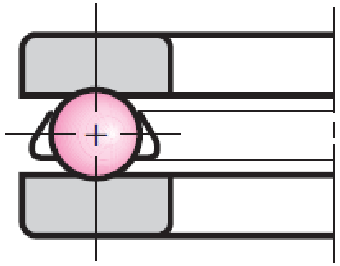


…

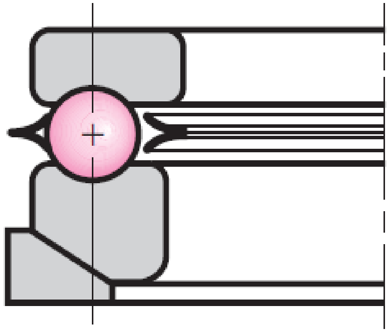
2. Thrust bearings

also called axial bearings

for carrying loads that are primarily axial.



Thrust



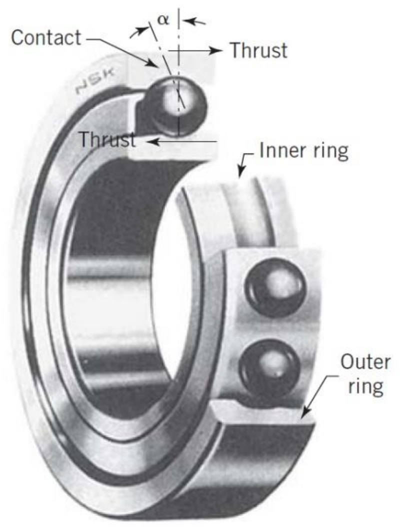
Self-aligning thrust

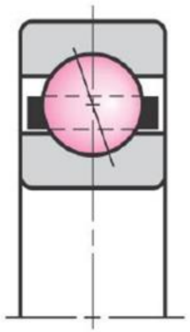
Thrust

…

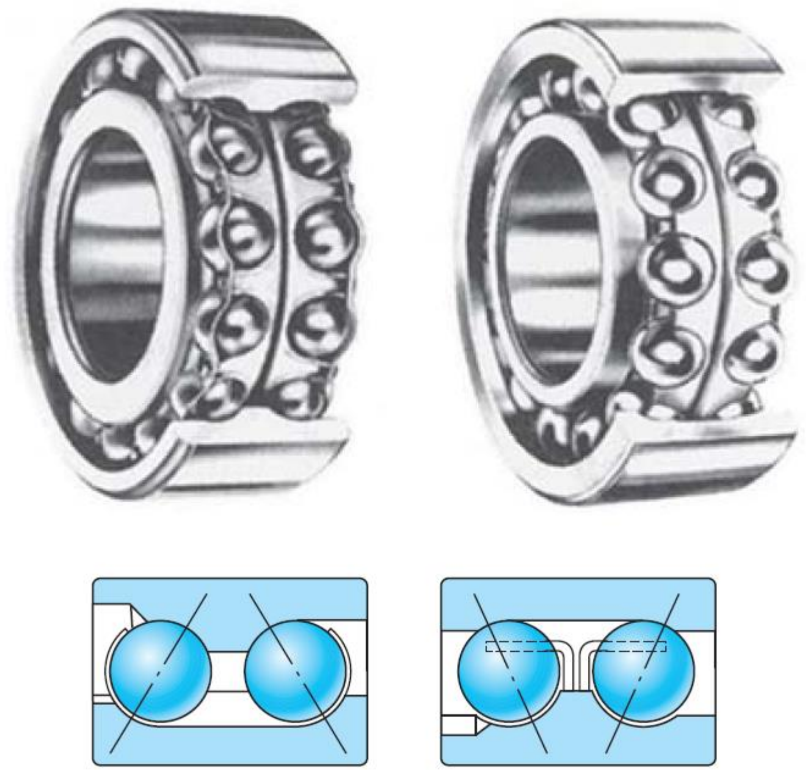
3. Angular-contact Bearings

for carrying combined axial and radial loads.





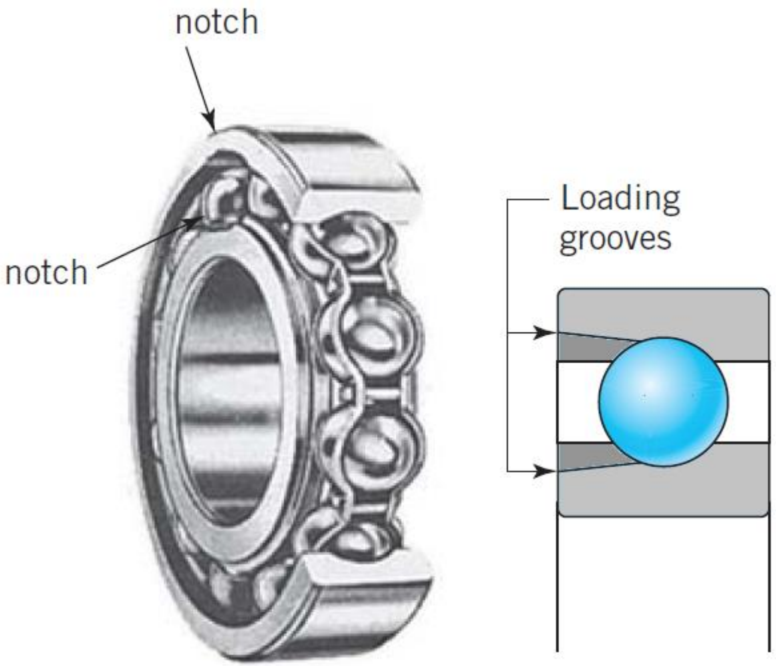
Angular contact



Double row

DEEP GROOVE BALL BEARINGS

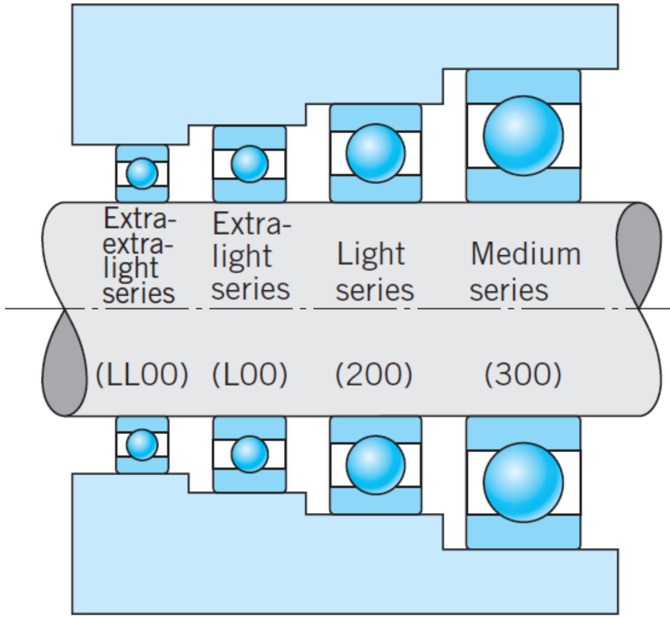
- *Deep-groove* ball bearings are intended primarily for **radial loads**.
- This is the most frequently used bearing found in a variety of machines.
- The radius of the ball is slightly less than the radii of curvature of the grooves in the races.
- Kinematically, this gives a point contact between the balls and the races



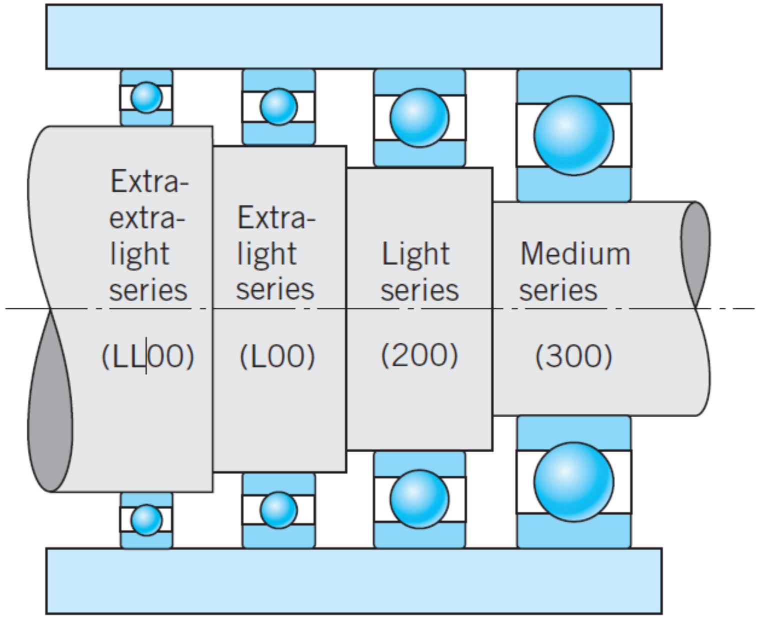
Contact surface geometry

....

- These ball bearings are made in various proportions for various degrees of loading.
- It is obvious from their construction that these bearings will also carry a certain amount of thrust.



(*a*)
Relative proportions of bearings with same bore dimension



(*b*)
Relative proportions of bearings with same outside diameter

…

Advantages of Deep Groove Ball Bearings

1. Due to relatively large size of the balls, their load carrying capacity is high.
2. Deep groove ball bearings are available with bore diameters from a few millimeters to 400 millimeters
3. Deep groove ball bearing generates less noise due to point contact.
4. Due to point contact between the balls and races, frictional loss and the resultant temperature rise is less.
5. The maximum permissible speed of the shaft depends upon the temperature rise, these bearings gives excellent performance, especially at high speeds.

....

Disadvantages of Deep Groove Ball Bearings

The disadvantages of deep groove ball bearings are as follows:

1. Deep groove ball bearing is not self aligning.
2. Accurate alignment between axes of the shaft and the housing bore is re.
3. Have poor rigidity as compared with roller bearing.

    This is due to the *point contact* compared with the *line contact* of roller bearing.

    Makes them unsuitable for machine tool spindles where rigidity is important consideration.

CYLINDRICAL ROLLER BEARINGS

- When the bearing has to carry high loads, the point contact is replaced by a line contact.

    This is done by using rollers in place of balls

    The bearing is then called a roller bearing.

- Cylindrical roller bearing consists of relatively short rollers with a separator or retainer to hold the rollers in place.

RADIAL

THRUST

....

Advantages of Cylindrical Roller Bearings

Cylindrical roller bearing offers the following advantages :

- Due to line contact between rollers and races, the load carrying capacity is very high
- Cylindrical roller bearing is more rigid than ball bearing.
- Frictional loss at high-speeds is lesser, due to lower coefficient of friction.

The **disadvantages** are as follows:

- (a) Cannot take thrust load.
- (b) They are not self-aligning, means the bearing cannot tolerate misalignment.

  (c) Generate more noise

SPHERICAL ROLLER BEARINGS

- The spherical-roller is useful where heavy loads and misalignment occur.
- The spherical elements have the advantage of increasing their contact area as the load is increased. There are of three types :
- The single-row type has little thrust capacity.
- The double-row type can carry thrust up to about 30% of its radial-load capacity.
- The angular-contact type

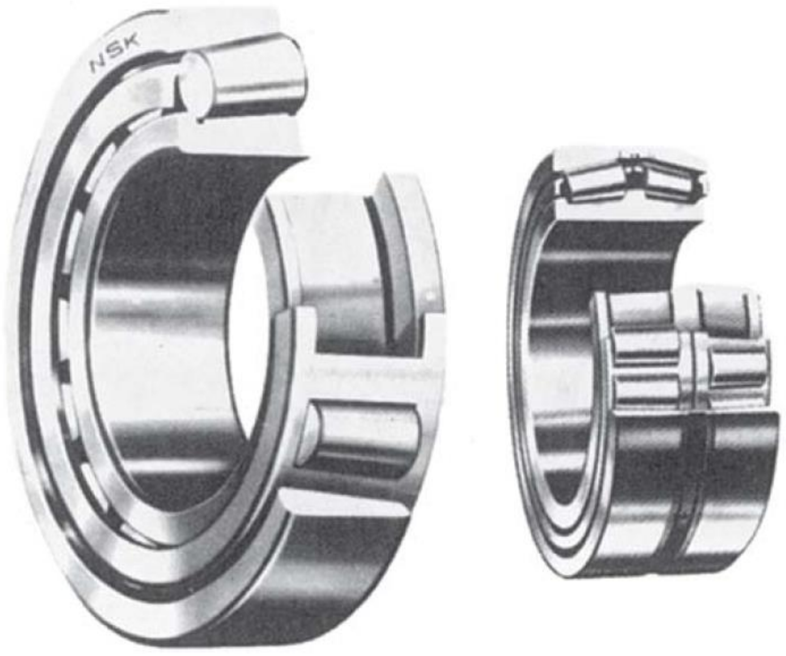
  can carry large thrust loads

  in one direction

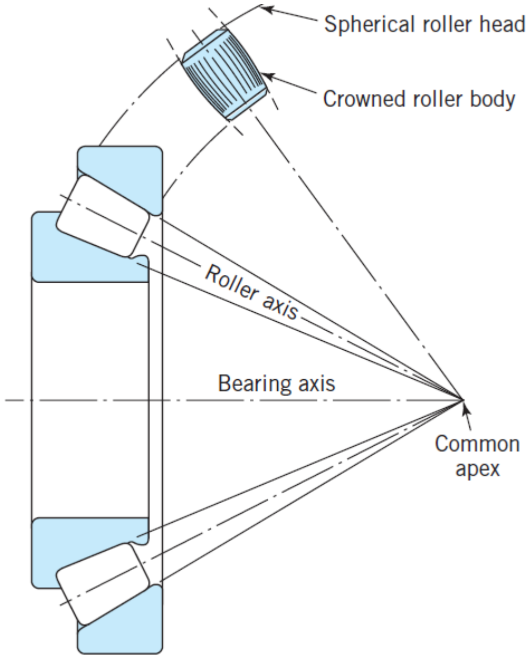
(a) Single-row convex (b) Double-row convex (c) Thrust

TAPERED ROLLER BEARINGS

- The rollers are now in the form of conical elements.
- All roller surfaces and the raceways intersect at a common point on the bearing centreline or rotation axis.



(*a*) Single-row (*b*) Double-row



Tapered roller bearing

...

- These bearings combine the advantages of ball and straight roller bearings: they can take either radial or thrust loads or any combination of the two,
- They have the high load-carrying capacity of straight rollers.
- Pairs of single-row tapered roller bearings are often used for wheel bearings.
- The double-row and four-row types replace single-row types for heavier loads.

NEEDLE BEARINGS

- Needle bearings can be regarded as a special case of cylindrical roller bearings.
- Here the rollers have a length-to-diameter ratio of four or greater, and are called needles.
- Because of their geometry, these bearings have for a given radial space the highest load capacity of all rolling-element bearings.

Radial
Caged type

Radial
Drawn Cup type

Thrust

- Drawn Cup type: bearings assembled without a retainer and with a full complement of rollers.

ANGULAR CONTACT BEARING

- In angular contact bearing, the grooves in inner and outer races are so shaped

  that the line of reaction at the contact between balls and races makes an angle with the axis of the bearing.

- These bearings, have substantial thrust capacity in one direction only.
- They are commonly installed in pairs, with each taking thrust in one direction.
- Angular contact bearings are often used in pairs, either side by side or at the opposite ends of the shaft,

  in order to take the thrust load in both directions.

- These bearings are assembled with a specific magnitude of preload.

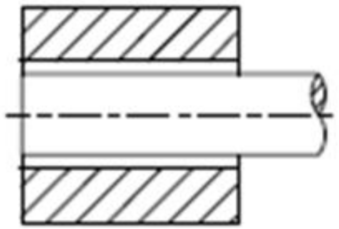
Advantages:

- Can take both radial and thrust loads.
- The load carrying capacity is more than that of deep groove ball bearing.

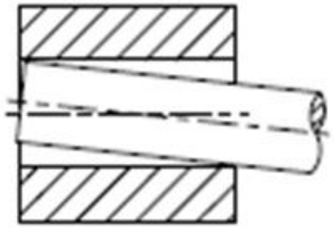
Disadvantages

- Two bearings are required to take thrust load in both directions.
- The angular contact bearing must be mounted without axial play.
- The angular contact bearing requires initial pre-loading

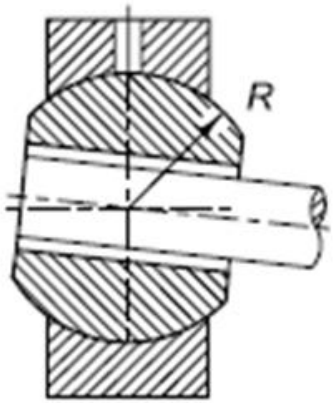
- The self-aligning ball bearing consists of two rows of balls, which roll on a common spherical surface in the outer race.
- In this case, the assembly of the shaft, the inner race and the balls with cage can freely roll and adjust itself to the angular misalignment of the shaft.



Shaft aligned with Bearing



Shaft misaligned with Bearing



Self aligning Bearing

....

- There is similar arrangement in the spherical roller bearing, where balls are replaced by two rows of spherical rollers, which run on a common spherical surface in the outer race.
- Compared with the self aligning ball bearing the spherical roller bearing can carry relatively high radial and thrust loads.
- Both types of self-aligning bearing permit minor angular misalignment of the shaft relative to the housing.
- They are therefore particularly suitable for applications where misalignment can arise due to errors in mounting or due to deflection of the shaft.
- They are used in agricultural machinery, ventilators, and railway axle-boxes.

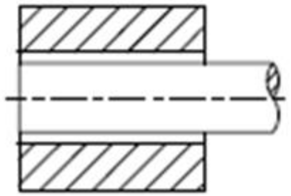
…

Two types of self-aligning bearings,

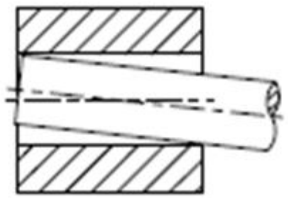
- self-aligning ball bearing and spherical roller bearing,
- In many applications, the bearing is required to tolerate a small amount of misalignment between the axes of the shaft and the bearing.
- The misalignment may be due to deflection of the shaft under load or due to tolerances of individual components.
- Self aligning bearings are used in these applications.
- The angular misalignment *a* is exaggerated in the figure. Self aligning bearings are commonly employed when accurate alignment is impossible or unfeasible.

....

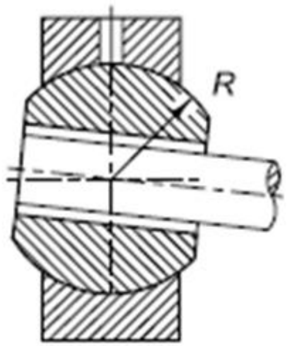
- The principle of self-aligning bearing is illustrated.
- The Centre of this spherical surface is at the Centre of the bearing. Therefore, the bush is free to roll in its seat and align itself with the journal.
- Arrangement is made to provide lubricant between the spherical surfaces of the bush and its seat in order to reduce the friction.
- This principle is used in self-aligning ball bearings and spherical roller bearings.



Shaft aligned with Bearing



Shaft misaligned with Bearing

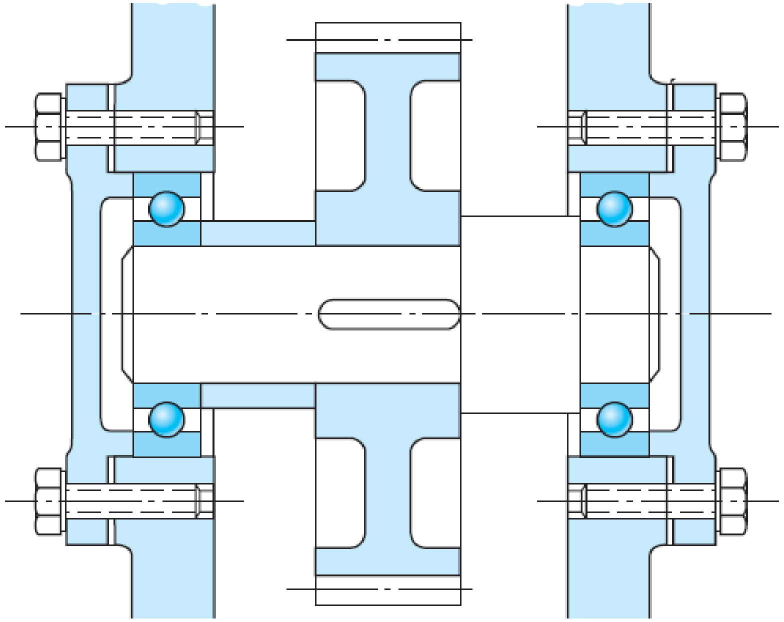


Self aligning Bearing

MACHINE DESIGN II

MEC 3110

LECTURE – 12



ROLLING-ELEMENT BEARINGS

PART – B

PROF. M. NAUSHAD ALAM

MECHANICAL ENGINEERING DEPARTMENT
A.M.U. ALIGARH

STATIC LOAD CARRYING CAPACITY OF A BEARING

- ***Static load***

  ***defied as the load acting on the bearing when the shaft is stationary*.**

  Static load produces permanent deformation in balls and races,
  which increases with increasing load.

  The permissible static load, therefore, depends upon the permissible magnitude of permanent deformation.

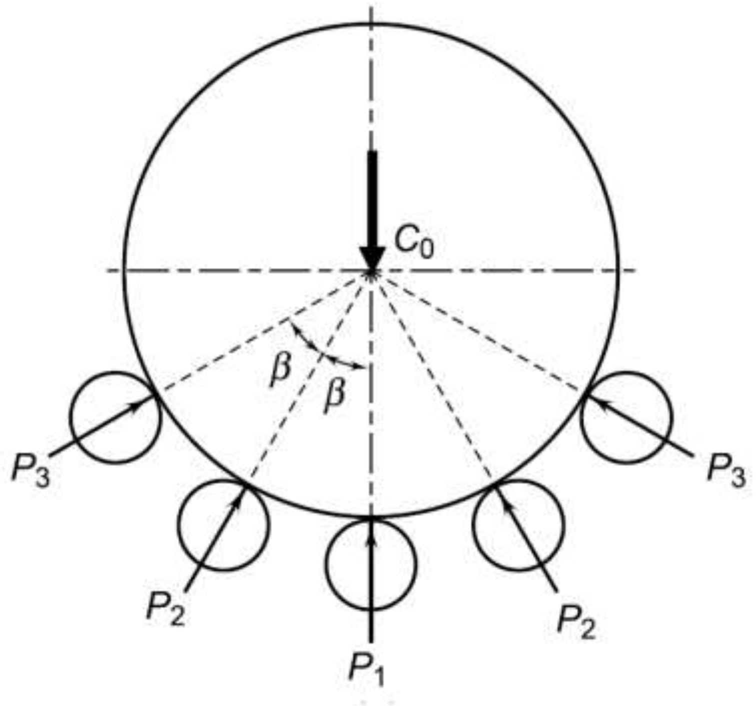
- ***The static load carrying capacity***

  ***defined as the load that corresponds to a total permanent deformation of balls and races a**t the most heavily stressed point of contact, equal to 0.0001 of the ball diameter*.

STRIBECK'S EQUATION

- Stribeck's equation gives the static load capacity of bearing.

- It is based on the following assumptions:
  - The races are rigid and retain their circular shape.
  - The balls are equally spaced.
  - The balls in the upper half do not support any load.
  - There is a single row of balls.



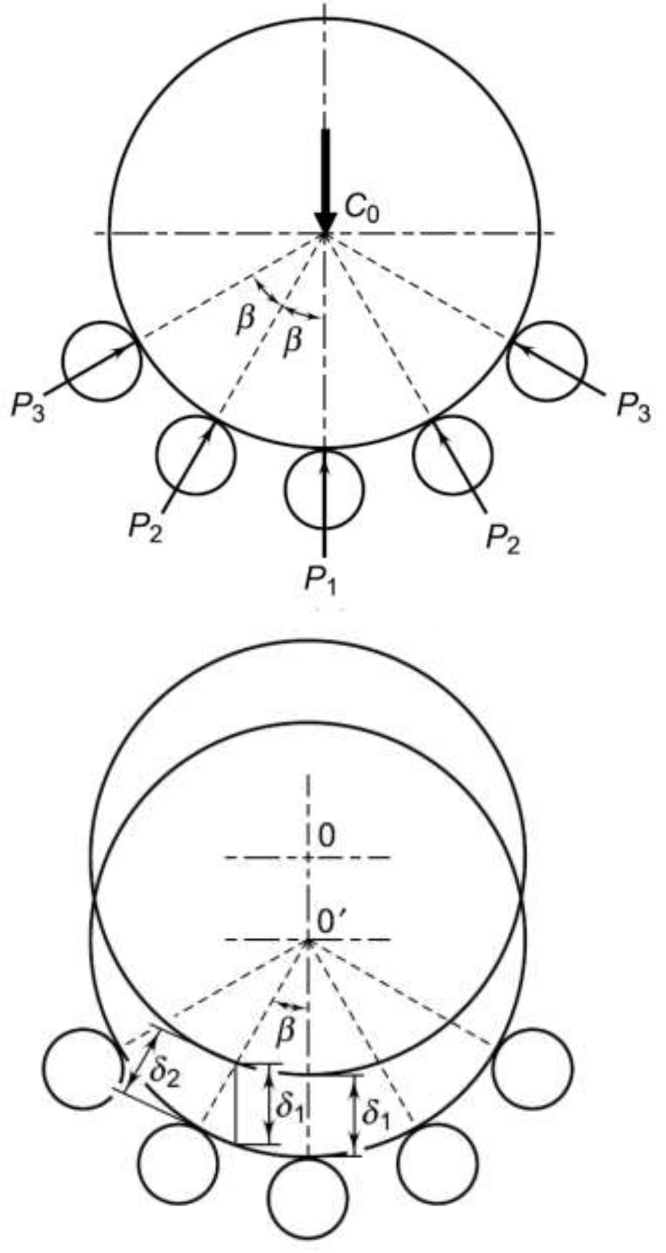
(a) Forces acting on Inner Race

...

- Assuming that only balls are deformed and considering the equilibrium of forces in the vertical direction,

$$C_o = P_1 + 2P_2 \cos\beta + 2P_3 \cos(2\beta) + \ldots\ldots \qquad \text{(a)}$$

- Suppose δ_1 is the deformation at the most heavily stressed Ball No.1.
- Due to this deformation, the inner race is deflected with respect to the outer race through δ_1
- The inner ring moves from O to O' through the distance without changing its shape



(b) Deflection of Inner Race

- Suppose $\delta_1, \delta_2 \ldots$ *are* radial deflections at the respective balls.
- Also $\delta_2 = \delta_1 \cos\beta$ or $\dfrac{\delta_2}{\delta_1} = \cos\beta$ (b)
- According to Hertz's equation,

  the bearing load and deflection at each ball are given by $\delta \propto (P)^{2/3}$

- Therefore, $\delta_1 = C_1 P_1^{2/3}$, $\delta_2 = C_1 P_2^{2/3}$ and $\dfrac{\delta_2}{\delta_1} = \left(\dfrac{P_2}{P_1}\right)^{2/3}$ ……. (c)
- From Equations (b) and (c),

$$\left(\frac{P_2}{P_1}\right)^{2/3} = \cos\beta$$

Or, $$P_2 = P_1 (\cos\beta)^{3/2}$$

$$C_o = P_1 + 2P_2 \cos \beta + 2P_3 \cos (2\beta) + \ldots\ldots \qquad \text{(a)}$$

- In a similar way,

$$P_3 = P_1 (\cos 2\beta)^{3/2}$$

- Substituting these values in Eq. (a),

$$C_0 = P_1 + 2[P_1(\cos \beta)^{3/2}] \cos \beta$$
$$+ 2[P_1(\cos 2\beta)^{3/2}] \cos 2\beta + \ldots\ldots$$

$$= P_1[1 + 2(\cos \beta)^{5/2} + 2(\cos 2\beta)^{5/2} + \ldots..]$$

$$\text{or} \quad C_o = P_1 M \qquad \text{(d)}$$

where

$$M = [1 + 2(\cos \beta)^{5/2} + 2(\cos 2\beta)^{5/2} + \ldots\ldots] \qquad \text{(e)}$$

- Where

$$M = [1 + 2(\cos\beta)^{5/2} + 2(\cos 2\beta)^{5/2} + ...] \qquad (e)$$

- If *z* is the number of balls.

$$\beta = \frac{360}{z}$$

- The values of *M* for different values of *z* are tabulated as follows:

Z	8	10	12	15
M	1.84	2.28	2.75	3.47
(z/M)	4.35	4.38	4.36	4.37

$$C_o = P_1 M \qquad (d)$$

- Tabulate values of *M*

Z	8	10	12	15
M	1.84	2.28	2.75	3.47
(z/M)	4.35	4.38	4.36	4.37

- It is seen from the above table that (*z*/*M*) is practically constant. Stribeck suggested the value for (*z*/*M*) as 5.
- Or, $M = \left(\frac{1}{5}\right) z$
- Substituting this value in Eq. (d), we get $C_o = \left(\frac{1}{5}\right) z P_1 \qquad (f)$

…

$$C_o = \left(\frac{1}{5}\right) z P_1 \qquad \text{(f)}$$

- From experimental evidence, it is found that the force *P***1** required to produce a given permanent deformation of the ball is given by,

$$P_1 = kd^2 \qquad \text{(g)}$$

where *d* is the ball diameter and the factor *k* depends upon the radii of curvature at the point of contact, and on the moduli of elasticity of materials.

- From Equations (f) and (g),

$$C_o = \frac{kd^2 z}{5}$$

Stribeck's equation.

LIFE OF BALL BEARING

- The ball bearing life is limited by the fatigue failure at the surfaces of balls and races.
- The dynamic load carrying capacity of the bearing is, therefore, based on the fatigue life of the bearing.

Bearing Life:

The life of an individual ball bearing is defined as the number of revolutions (or hours of service at some given constant speed), which the bearing runs before the first evidence of fatigue crack in balls or races.

Life of Ball Bearing

- The life of a single bearing is difficult to predict,

- It is therefore necessary to define the bearing life in terms of the statistical average performance for a group of bearings.

- Bearings are rated on one of the two criteria—
  - The average life of a group of bearings
  - The life which 90% of the bearings will reach or exceed.
  - The second criteria is widely used in bearing industry

Life of Ball bearing

Rating life :

The rating life of a group of apparently identical ball bearings is defined as the number of revolutions that 90% of the bearings will complete or exceed before the first evidence of fatigue crack.

- There are a number of terms used for the rating life:

  minimum life,

  catalogue life,

  $L10$ life or $B10$ life.

  The terms are synonyms for rating life, the term $L10$ life is commonly used.

- The life of an individual ball bearing may be different from rating life.

DYNAMIC LOAD CARRYING CAPACITY

Dynamic load carrying capacity of a bearing

It is defined as the radial load in radial bearings (or thrust load in thrust bearings) that can be carried for a minimum life of one million revolutions.

- The minimum life in this definition is the $L10$ life, which 90% of the bearings will reach or exceed before fatigue failure.
- The dynamic load carrying capacity is based on the assumption that the inner race is rotating while the outer race is stationary.
- The formulae for calculating the dynamic load capacity for different types of bearings are given in standards.
- However, the manufacturer's catalogues give ready-made values of dynamic load capacities of bearings.

EQUIVALENT BEARING LOAD

- In actual applications, the force acting on the bearing has two components —radial and thrust.

Equivalent dynamic load

The equivalent dynamic load is defined as the constant radial load in radial bearings (or thrust load in thrust bearings), which if applied to the bearing would give same life as that which the bearing will attain under actual condition of forces.

- The expression for the equivalent dynamic load is written as, $P = XVF_r + YF_a$

where, P = equivalent dynamic load (N)

F_r = radial load (N)

F_a = axial or thrust load (N)

V = race-rotation factor

X and Y are radial and thrust factors respectively and their values are given in the manufacturer's catalogues.

Equivalent Bearing Load $P = XVF_r + YF_a$

- V = race-rotation factor
  - It depends upon whether the inner race is rotating or the outer race.
  - The value of V is 1 when the inner race rotates while the outer race is held stationary in the housing.
  - The value of V is 1.2 when the outer race rotates with respect to the load, while the inner race remains stationary.
- In most of the applications, the inner race rotates and the outer race is fixed in the housing.
- Assuming V as unity, the general equation for equivalent dynamic load is given by,

$$P = XF_r + YF_a$$

- When the bearing is subjected to pure radial load *Fr,*
- When the bearing is subjected to pure thrust load *Fa* ,

BEARING LIFE DEFINITIONS

- ***Bearing Failure:*** Spalling or pitting of an area of 0.01 in**2**

- ***Life***: Number of revolutions (or hours @ given speed) required for failure.
  - For one bearing

- ***Rating Life***: *Life* required for 10% of sample to fail.
  - For a group of bearings
  - Also called *Minimum Life* or ***L10*** *Life*

- ***Median Life***: Average life required for 50% of sample to fail.
  - For many groups of bearings
  - Also called *Average Life* or *Average Median Life*
  - *Median Life* is typically 4 or 5 times the *L10 Life*

LOAD-LIFE RELATIONSHIP

- The relationship between the dynamic load carrying capacity, the equivalent dynamic load, and the bearing life is given by,

$$L_{10} = \left(\frac{C}{P}\right)^p$$

L_{10} = rated bearing life (in million revolutions)

C = dynamic load capacity (N), and

p = 3 (for ball bearings)

p = 10/3 (for roller bearings)

- The relationship between life in million revolutions and life in working hours is given by

$$L_{10} = \frac{60nL_{10h}}{10^6}$$

- Where,

  L_{10h} = rated bearing life (hours)

  n = speed of rotation (rpm)

End of Lecture

MACHINE DESIGN II

MEC 3110

LECTURE - 13

ROLLING-ELEMENT BEARING PROBLEMS

PROF. M. NAUSHAD ALAM

MECHANICAL ENGINEERING DEPARTMENT
A.M.U. ALIGARH

Problem-1

In a particular application, the radial load acting on a ball bearing is 5 kN and the expected life for 90% of the bearings is 8000 h. Calculate the dynamic load carrying capacity of the bearing, when the shaft rotates at 1450 rpm.

Solution:

Given Data: $F_r = 5\text{ kN}$ $\quad L_{10h} = 8000\text{ h}$ $\quad n = 1450\text{ rpm}$

Step I *Bearing life (L_{10})*

$$L_{10} = \frac{60nL_{10h}}{10^6}$$

$$= \frac{60(1450)(8000)}{10^6}$$

$$= 696 \text{ million rev.}$$

Step II *Dynamic load capacity*

$P = F_r = 5000\text{ N}$ purely radial load,

$$C = P(L_{10})^{1/3}$$

$$= (5000)(696)^{1/3}$$

$$= 44\,310.48\text{ N}$$

Problem-2

A taper roller bearing has a dynamic load capacity of 26 kN. The desired life for 90% of the bearings is 8000 h and the speed is 300 rpm. Calculate the equivalent radial load that the bearing can carry.

Solution:

Given; ***$C = 26$ kN, $L_{10h} = 8000$ h, $n = 300$ rpm***

Step I *Bearing life (L_{10})*

$$L_{10} = \frac{60nL_{10h}}{10^6}$$

$$= \frac{60(300)(8000)}{10^6}$$

$= 144$ million rev.

Step II *Equivalent radial load*

$$C = P\,(L_{10})^{0.3}$$

$$\therefore \quad P = \frac{C}{(L_{10})^{0.3}}$$

$$= \frac{26\,000}{(144)^{0.3}} = 5854.16 \text{ N}$$

SELECTION OF BEARING LIFE

- While selecting the proper size of a bearing, it is necessary to specify the expected life of the bearing for the given application.
- The information regarding the life expectancy is generally vague and values based on past experience are used.
- For all kinds of vehicles, the speed of rotation is not constant and the desired life is expressed in terms of millions of revolutions. The recommended bearing life for wheel applications is given in Table below.

Bearing life for wheel applications

Wheel application	*Life (million rev.)*
Automobile cars	50
Trucks	100
Trolley cars	500
Rail-road cars	1000

Selection of Bearing Life

- In the other applications, the speed of rotation is relatively constant and the desired life is expressed in terms of hours of service.

Bearing life for industrial applications

(i) Machines used intermittently such as lifting tackle, hand tools and household appliances	4000–8000 h
(ii) Machines used for eight hours of service per day, such as electric motors and gear drives	12 000–20 000 h
(iii) Machines used for continuous operation (24 h per day) such as pumps, compressors and conveyors	40 000–60 000 h

LOAD FACTOR

- The forces acting on the bearing are calculated by considering the equilibrium of forces in vertical and horizontal planes.
- These elementary equations do not take into consideration the effect of dynamic load.
- The forces determined by these equations are multiplied by a load factor to determine the dynamic load carrying capacity of the bearing.
- Load factors are used in applications involving gear, chain and belt drives.

Values of load factor

Types of drive	*Load factor*
(A) *Gear drives*	
(i) Rotating machines free from impact like electric motors and turbo-compressors	1.2–1.4
(ii) Reciprocating machines like internal combustion engines and compressors	1.4–1.7
(iii) Impact machines like hammer mills	2.5–3.5
(B) *Belt drives*	
(i) V-belts	2.0
(ii) Single-ply leather belt	3.0
(iii) Double-ply leather belt	3.5
(C) *Chain drives*	1.5

SELECTION OF BEARING

Selection of a bearing from the manufacturer's catalogue

The basic procedure for the selection consists of the following steps:

(i) Calculate the radial and axial forces acting on the bearing and determine the diameter of the shaft where the bearing is to be fitted.

(ii) Select the type of bearing for the given application.

(iii) Determine the values of X and Y, the radial and thrust factors, from the catalogue. The values of X and Y factors for single-row deep groove ball bearings are given in Table.

The values depend upon two ratios $\left(\dfrac{F_a}{F_r}\right)$ and $\left(\dfrac{F_a}{C_0}\right)$,

where C_0 is the static load capacity.

(iv) Calculate the equivalent dynamic load from the equation.

$$P = XF_r + YF_a$$

(v) Make a decision about the expected bearing life and express the life $L10$ in million revolutions.

(vi) Calculate the dynamic load capacity from the equation.

$$C = P\,(L_{10})^{1/3}$$

(vii) Check whether the selected bearing of series 60 has the required dynamic capacity. If not, select the bearing of the next series and go back to Step (iii) and continue.

Ball bearings are thus selected by the trial and error procedure. The above procedure is also applicable to other types of bearings.

Table

Table 15.4 *X and Y factors for single-row deep groove ball bearings*[3]

$\left(\frac{F_a}{C_0}\right)$	$\left(\frac{F_a}{F_r}\right) \leq e$		$\left(\frac{F_a}{F_r}\right) > e$		e
	X	Y	X	Y	
0.025	1	0	0.56	2.0	0.22
0.040	1	0	0.56	1.8	0.24
0.070	1	0	0.56	1.6	0.27
0.130	1	0	0.56	1.4	0.31
0.250	1	0	0.56	1.2	0.37
0.500	1	0	0.56	1.0	0.44

PROBLEM-3

Select a single-row deep groove ball bearing, for a shaft that is 75 mm in diameter and which rotates at 125 rpm. The bearing is subjected to a radial load of 21 kN and there is no thrust load. The expected life of the bearing is 10 000 hours.

Solution

Given,

$F_r = 21\,000$ N, $F_a = 0$ $d = 75$ mm

Type: single-row deep groove ball bearing

Step (iii) Since there is no axial load,

$$P = F_r = 21\,000 \text{ N}$$

Solution

$$L_{10} = \frac{60n\, L_{10h}}{10^6} = \frac{60\,(125)(10\,000)}{10^6}$$

$$= 75 \text{ million rev.}$$

$$C = P\,(L_{10})^{1/3} = 21\,000\,(75)^{1/3}$$

$$= 88\,560.43 \text{ N}$$

Step (vi) It is observed from Table 15.5, that following bearings are available with 75mmbore diameter,

...

Table 15.5 *Dimensions and static and dynamic lo[ad] capacities of single-row deep groove ball bearings*[4]

Principal dimensions (mm)			Basic load ratings (N)		Designation
d	D	B	C	C_0	
10	19	5	1480	630	61800
	26	8	4620	1960	6000
	30	9	5070	2240	6200
	35	11	8060	3750	6300

(Cont[d])

d = inner diameter of the bearing
D = outer diameter of the bearing
B = axial width of the bearing

Table 15.5 (Contd)

Principal dimensions (mm)			Basic load ratings (N)		Designation
d	D	B	C	C_0	
12	21	5	1430	695	61801
	28	8	5070	2240	6001
	32	10	6890	3100	6201
	37	12	9750	4650	6301
15	24	5	1560	815	61802
	32	9	5590	2500	6002
	35	11	7800	3550	6202
	42	13	11400	5400	6302
17	26	5	1680	930	61803
	35	10	6050	2800	6003
	40	12	9560	4500	6202
	47	14	13500	6550	6303
	62	17	22900	11800	6403

Table 15.5 *(Contd)*

Principal dimensions (mm)			*Basic load ratings (N)*		*Designation*
d	*D*	*B*	*C*	C_0	
70	90	10	12100	9150	61814
	110	13	28100	19000	16014
	110	20	37700	24500	6014
	125	24	61800	37500	6214
	150	35	104000	63000	6314
	180	42	143000	104000	6414
75	95	10	12500	9800	61815
	115	13	28600	20000	10615
	115	20	39700	26000	6015
	130	25	66300	40500	6215
	160	37	112000	72000	6315
	190	45	153000	114000	6415

…

- A rolling contact bearing is usually designated by three or four digits. The meaning of these digits is as follows:

  (i) The last two digits indicate the bore diameter of the bearing in mm (bore diameter divided by 5). For example, XX07 indicates a bearing of 35 mm bore diameter.

  (ii) The third digit from the right indicates the series of the bearing. The numbers used to indicate the series are as follows:

Extra light series –1	Light series – 2
Medium series – 3	Heavy series – 4

  For example, X307 indicates a medium series bearing with a bore diameter of 35 mm.

  (iii) The fourth digit and sometimes fifth digit from the right specifies the type of rolling contact bearing. For example, the digit 6 indicates deep groove ball bearings

PROBLEM

A single-row deep groove ball bearing is subjected to a radial force of 8 kN and a thrust force of 3 kN. The shaft rotates at 1200 rpm. The expected life L10h of the bearing is 20 000 h. The minimum acceptable diameter of the shaft is 75 mm. Select a suitable ball bearing for this application

Solution

Given $F_r = 8$ kN, $F_a = 3$ kN, $L_{10h} = 20\,000$ hr

$n = 1200$ rpm, $d = 75$ mm

Step I *X and Y factors*

When the bearing is subjected to radial as well as axial load, the values of *X* and Y factors are obtained from Table 15.4 by trial and error procedure. It is observed from Table 15.4, that values of X are constant and the values of Y vary only in case when

$$\left(\frac{F_a}{F_r}\right) > e$$

- In this case, the value of *Y* varies from 1.0 to 2.0.
- Assume the average value 1.5 as the first trial value for the factor *Y*. Therefore,

$$X = 0.56 \quad Y = 1.5 \quad Fr = 8000 \text{ N} \quad Fa = 3000 \text{ N}$$

$$P = XFr + YFa = 0.56(8000) + 1.5(3000) = 8980 \text{ N}$$

$$L_{10} = \frac{60n\,L_{10h}}{10^6} = \frac{60(1200)(20\,000)}{10^6}$$

$$= 1440 \text{ million rev.}$$

- C = P (L10)1/3 = (8980)(1440)1/3 = 101 406.04 N

,,

- From Table 15.5, it is observed that for the shaft of 75 mm diameter, Bearing No. 6315 (*C* = 112 000) is suitable for the above data. For this bearing, *Co* = 72 000 N

$$\left(\frac{F_a}{F_r}\right) = \left(\frac{3000}{8000}\right) = 0.375$$

$$\left(\frac{F_a}{C_o}\right) = \left(\frac{3000}{72000}\right) = 0.04167$$

- e = 0.24 (approximately

The value of Y is obtained by linear interpolation.

$$Y = 1.8 - \frac{(1.8 - 1.6)}{(0.07 - 0.04)} \times (0.04167 - 0.04) = 1.79$$

and $\quad X = 0.56$

Problem

A transmission shaft rotating at 720 rpm and transmitting power from the pulley P to the spur gear G is shown in Fig. The belt tensions and the gear tooth forces are as follows:

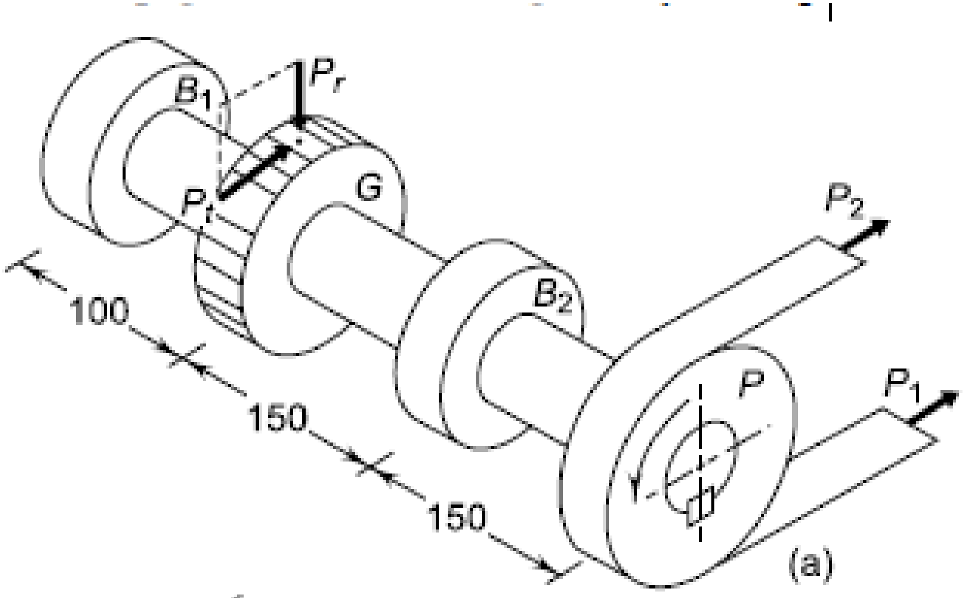
$P_1 = 498$ N $P_2 = 166$ N $P_t = 497$ N $P_r = 181$ N

The weight of the pulley is 100 N. The diameter of the shaft at bearings B1 and B2 is 10 mm and 20 mm respectively. The load factor is 2.5 and the expected life for 90% of the bearings is 8000 h. Select single row deep groove ball bearings at B1 and B2.

Solution

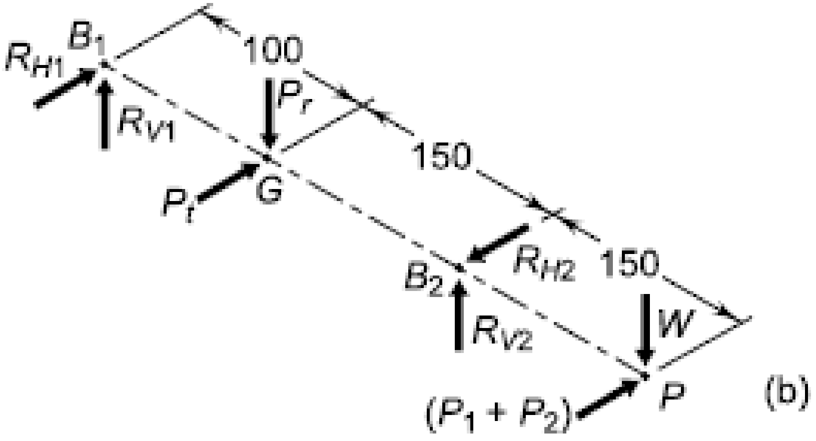
Given :

$n = 720$ rpm,
$d1 = 10$ mm,
$d2 = 20$ mm
$L10h = 8000$ h
load factor $= 2.5$



Step I *Radial and axial forces*

Considering forces acting on the shaft in the vertical plane



$$R_{V1} + R_{V2} = P_r + W$$

or $\quad R_{V1} + 232.4 = 181 + 100$

$\therefore \quad R_{V1} = 48.6$ N

Solution

- Considering forces in the horizontal plane and taking moments of forces about the bearing B1,

$$P_t(100) + (P_1 + P_2)(400) - R_{H2}(250) = 0$$

or $$497(100) + (498 + 166)(400) - R_{H2}(250) = 0$$

$\therefore$ $$R_{H2} = 1261.2 \text{ N}$$

$$R_{H2} = R_{H1} + P_t + (P_1 + P_2)$$

or $$1261.2 = R_{H1} + 497 + (498 + 166)$$

$\therefore$ $$R_{H1} = 100.2 \text{ N}$$

- The reactions at the two bearings are given by,

$$R_1 = \sqrt{(R_{V1})^2 + (R_{H1})^2} = \sqrt{(48.6)^2 + (100.2)^2}$$

$$= 111.36 \text{ N}$$

$$R_2 = \sqrt{(R_{V2})^2 + (R_{H2})^2} = \sqrt{(232.4)^2 + (1261.2)^2}$$

$$= 1282.43 \text{ N}$$

- The bearing reactions are in the radial direction.

$$F_{r1} = R_1 = 111.36 \text{ N}$$

$$F_{r2} = R_2 = 1282.43 \text{ N}$$

- There is no axial thrust on these bearings; hence,

$$F_{a1} = F_{a2} = 0$$

Solution

Step II ***Dynamic load capacities***

Considering forces acting on the shaft

$$P_1 = F_{r1} = 111.36\text{ N}$$

$$P_2 = F_{r2} = 1282.43\text{ N}$$

$$L_{10} = \frac{60nL_{10h}}{10^6} = \frac{60(720)(8000)}{10^6}$$

$$= 345.6\text{ million rev.}$$

Considering the load factor

$$C_1 = P_1(L_{10})^{1/3}\text{ (Load factor)}$$

$$= (111.36)(345.6)^{1/3}(2.5)$$

$$= 1953.71\text{ N}$$

$$C_2 = P_2(L_{10})^{1/3}\text{ (Load factor)}$$

$$= (1282.43)(345.6)^{1/3}(2.5)$$

$$= 22499.09\text{ N}$$

Solution

Step III *Selection of bearings*

From the Table, the following bearings are available for 10 mm and 20 mm shaft diameter;

Bearing Nos. 6000
and 6404
are suitable at B_1 and B_2 respectively.

Dimensions and static and dynamic load capacities of single-row deep groove ball bearings

Principal dimensions (mm)			*Basic load ratings (N)*		*Designation*
d	*D*	*B*	*C*	C_0	
10	19	5	1480	630	61800
	26	8	4620	1960	6000
	30	9	5070	2240	6200
	35	11	8060	3750	6300

(*Contd*)

PROBLEM

A shaft transmits 50 kW at 125 rpm from the gear G1 to the gear G2 and mounted on Rolling Contact Bearings 579 two single-row deep groove ball bearings B1 and B2. The gear tooth forces are

$$P_{t1} = 15915 \text{ N} \quad P_{r1} = 5793 \text{ N}$$

$$P_{t2} = 9549 \text{ N} \quad P_{r2} = 3476 \text{ N}$$

The diameter of the shaft at bearings B1 and B2 is 75 mm. The load factor is 1.4 and the expected life for 90% of the bearings is 10000 h. Select suitable ball bearings.

Solution

Given

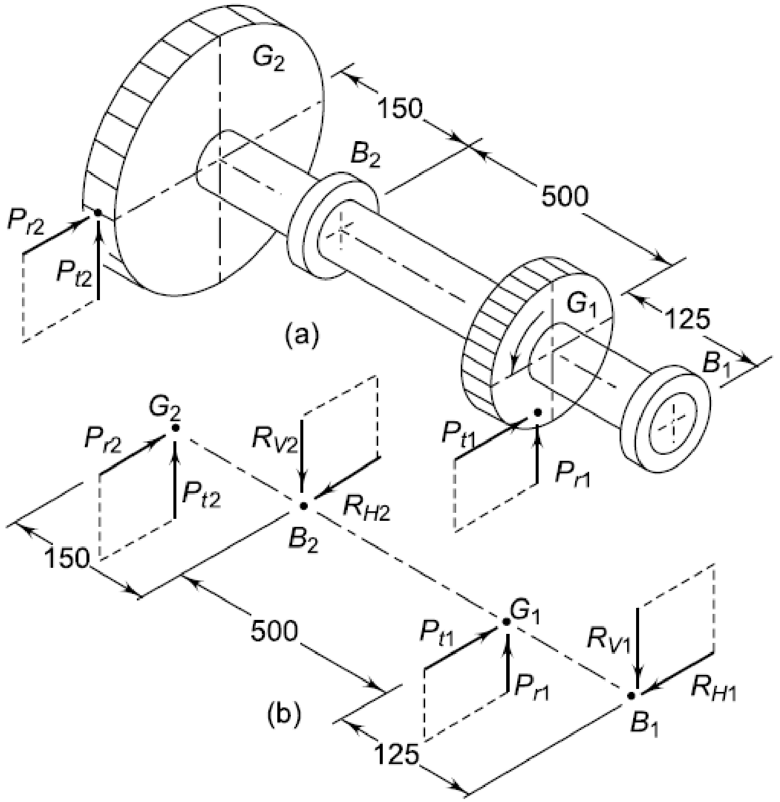
$$kW = 50$$
$$n = 125 \text{ rpm}$$
$$d = 75 \text{ mm}$$
$$L10h = 10\,000 \text{ h}$$
$$\text{load factor} = 1.4$$

Considering forces in the vertical plane and taking moments about bearing *B*1,

$$P_{r1}(125) + P_{r2}(775) - R_{V2}(625) = 0$$

or $\quad 5793(125) + 9549(775) - R_{V2}(625) = 0$

$\therefore \quad R_{V2} = 12\,999 \text{ N}$



Considering equilibrium of forces in the vertical plane

$$P_{t2} + P_{r1} = R_{v2} + R_{v1}$$

$$9549 + 5793 = 12\,999 + R_{V1}$$

$$R_{V1} = 2343 \text{ N}$$

A similar procedure is repeated for calculating forces in the horizontal plane and the reactions

$$R_{H1} = 11\,898 \text{ N} \qquad R_{H2} = 7493 \text{ N}$$

The radial forces at the two bearings are given by

$$F_{r1} = \sqrt{(R_{V1})^2 + (R_{H1})^2} = \sqrt{(2343)^2 + (11898)^2}$$

$$= 12\,127 \text{ N}$$

Since there is no axial thrust, $F_{a1} = F_{a2} = 0$

Step II *Dynamic load capacities*

Considering forces acting on the shaft in the vertical plane

$$P_1 = F_{r1} = 12\,127 \text{ N}$$

$$P_2 = F_{r2} = 15\,004 \text{ N}$$

$$L_{10} = \frac{60nL_{10h}}{10^6} = \frac{60(125)(10000)}{10^6}$$

$$= 75 \text{ million rev.}$$

Considering the load factor and using the dynamic load capacities are given by,

$$C_1 = P_1(L_{10})^{1/3} \text{ (Load factor)}$$

$$= (12127)(75)^{1/3}(1.4)$$

$$= 71\,598 \text{ N}$$

$$C_2 = P_2(L_{10})^{1/3} \text{ (Load factor)}$$

$$= (15004)(75)^{1/3}(1.4) = 88\,584 \text{ N}$$

...

Step III *Selection of bearings*

From the Table,

the available bearings at B_1 and B_2 are as follows:

B_1 and B_2 ($d = 75$ mm)

No. 6015 ($C = 39\,700$ N)

No. 6215 ($C = 66\,300$ N)

No. 6315 ($C = 112\,000$ N)

No. 6415 ($C = 153\,000$ N)

Dimensions and static and dynamic load capacities of single-row deep groove ball bearings

Principal dimensions (mm)			*Basic load ratings (N)*		*Designation*
d	*D*	*B*	*C*	C_0	
10	19	5	1480	630	61800
	26	8	4620	1960	6000
	30	9	5070	2240	6200
	35	11	8060	3750	6300

(*Contd*)

- Therefore, Bearing No. 6315 is suitable at B_1 as well as B_2.

Step IV ***Bearing life (L50)***

It can be proved that the life (*L*50), which 50% of the bearings will complete or exceed, is approximately five times the life *L*10 which 90% of the bearings will complete or exceed.

- Therefore,.

$$L_{50} = 5L_{10} = 5\,(14.13) = 70.65 \text{ million rev.}$$

End of Lecture

Lecture - 14

Rolling-Element Bearings

By

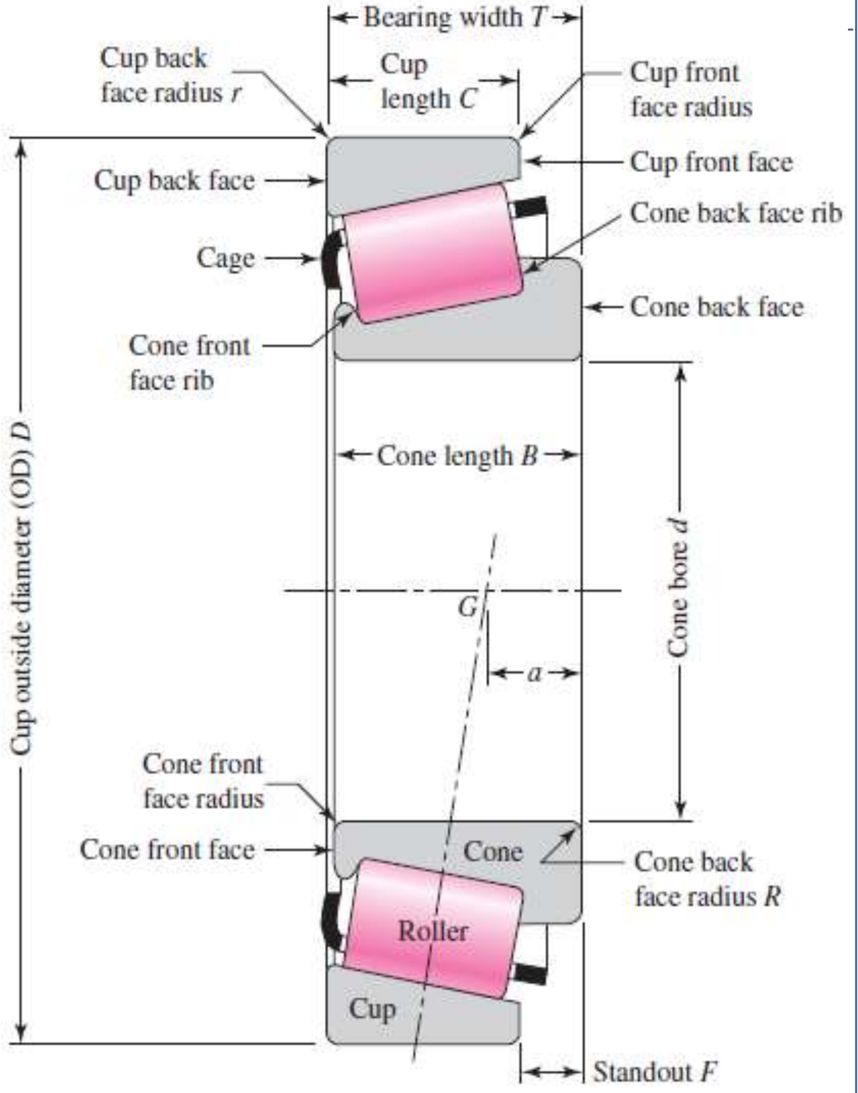
Prof. M. Naushad Alam

MACHINE DESIGN II
MEC 3110

MECHANICAL ENGINEERING DEPT.
A.M.U. ALIGARH

Tapered Roller Bearings

- Straight roller bearings can carry large radial loads, but, cant carry axial loads.
- Ball bearings can carry moderate radial loads, and small axial loads.
- For combined radial and axial thrust loads, tapered roller bearings provide a good solution.
- A tapered roller bearing can carry
  - Radial loads
  - Thrust (axial) loads,
  - or any combination of the above two loads



Tapered Roller Bearings

Tapered roller bearing has a separable construction.

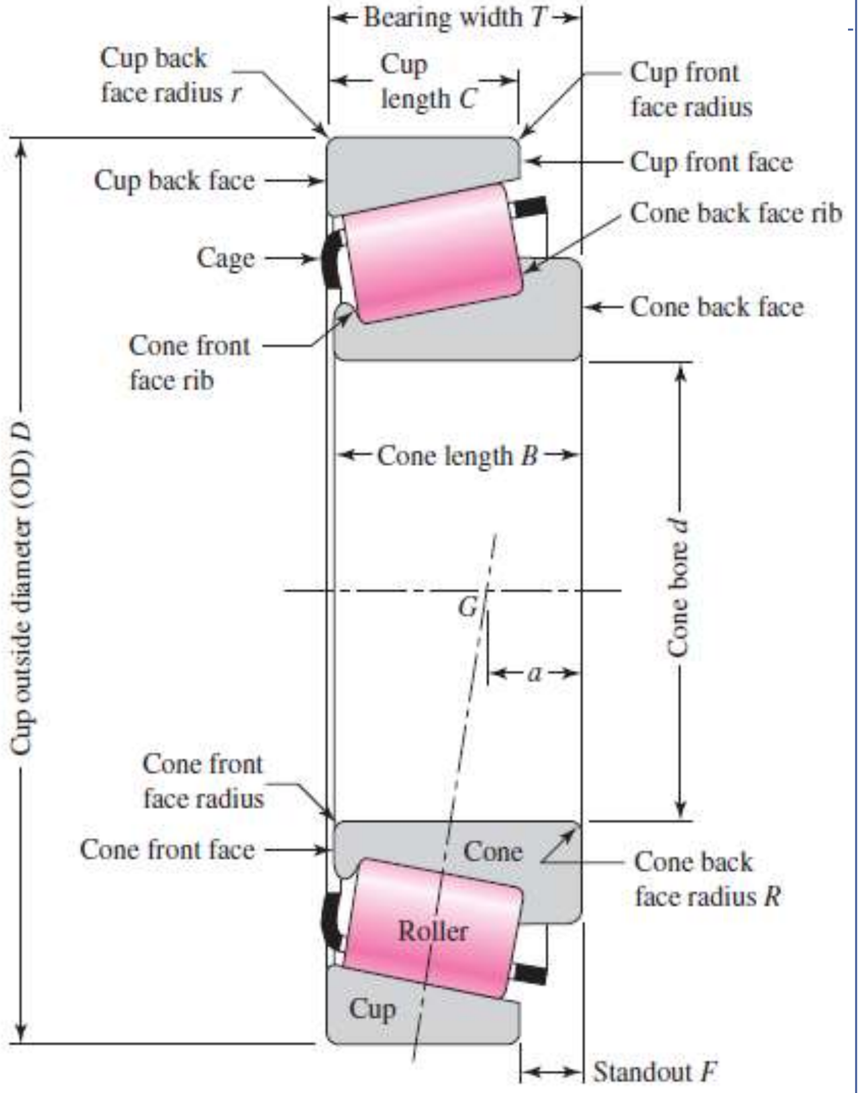
The assembled bearing consists of two separable parts:

1. The outer ring called the Cup

   It is separable from the remainder assembly of the bearing elements comprising the cone.

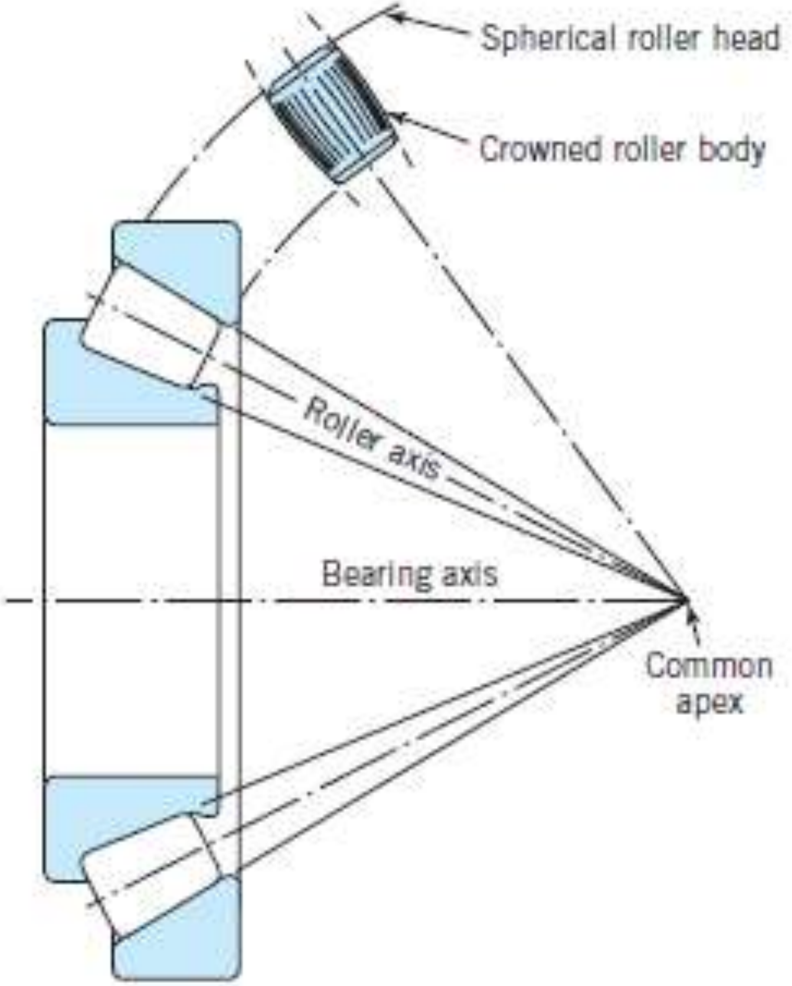
2. The the inner ring is called ‘cone assembly consists of
   - the cone,
   - the rollers,
   - and the cage

   Distance a locates the effective axial location for force analysis.



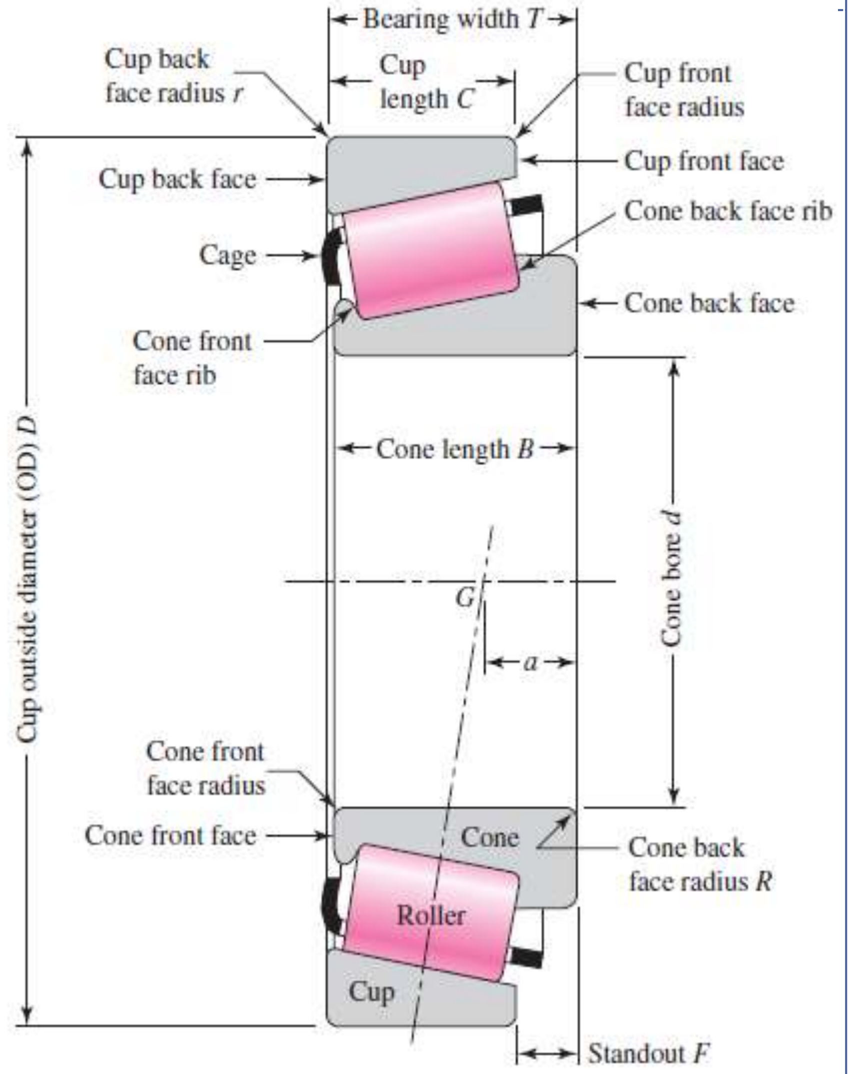
Tapered Roller Bearings

- Rollers are tapered so virtual apex is on shaft centerline.
- Taper allows for pure rolling of angled rollers.
- The rolling elements are in the form of a frustum cone.
- They are arranged in such a way that the axes individual rolling elements intersect at a common ap point on the axis of the bearing.
- In kinematic analysis, this is the essential requirem for pure rolling motion between conical surfaces.



Tapered Roller Bearings

- In tapered roller bearings, the line of resultant reaction through the rolling elements makes an angle with the axis of the bearing.
- Therefore, these bearings can carry both radial and axial loads.
- In fact, the presence of either component results in the other, acting on the bearing.
- In other words, a taper roller bearing subjected to pure radial load induces a thrust component and vice versa.
- Therefore, taper roller bearings are always used in pairs to balance the thrust component.



Advantages and Disadvantages of taper roller bearing

Advantages:

Taper roller bearings offer the following advantages:

- Can take heavy radial and thrust loads.
- Have more rigidity.
- The bearing can be easily assembled and disassembled due to separable parts.

Disadvantages:

- Taper roller bearings have the following disadvantages:
- It is necessary to use two taper roller bearings to balance the axial forces.
- It is necessary to adjust the axial position of the bearing with pre-load.

  This is essential to coincide apex of cone with the common apex of the rolling elements.
- Cannot tolerate misalignment between the axes of the shaft and the housing bore.
- Taper roller bearings are costly.

Applications of taper roller bearing

Taper roller bearings have the following applications:

- Cars,
- Trucks,
- Propeller shafts,
- Differentials,
- Railroad axle-boxes
- Large size bearings in rolling mills.

DESIGN FOR CYCLIC LOADS AND SPEEDS

In certain applications, ball bearings are subjected to cyclic loads and speeds

- radial load 2500 N at 700 rpm for 25% of the time,
- radial load 5000 N at 900 rpm for 50% of the time, and
- radial load 1000 N at 750 rpm for the remaining 25% of the time.

- Under these circumstances, it is necessary to consider the complete work cycle while finding out the dynamic load capacity of the bearings.

- The procedure for determining the dynamic load capacity of the bearing. consists of dividing the work cycle into a number of elements, while the operating conditions of load and speed are constant.

Design For Cyclic Loads and Speeds

- Suppose that the work cycle is divided into *x* elements.

  Let P_1, P_2, … P_x be the loads

  and n_1, n_2,…, n_x be the speeds during these elements.

- During the first element, the life $L1$ corresponding to load $P1$, is given by

$$L_1 = \left(\frac{C}{P_1}\right)^3 \times 10^6 \text{ rev.}$$

- In one revolution, the life consumed is $\left(\frac{1}{L_1}\right)$ or $\left(\frac{P_1^3}{C^3} \times \frac{1}{10^6}\right)$

- Let us assume that the first element consists of N**1** revolutions. Therefore, the life consumed by the first element is given by,

$$\frac{N_1 P_1^3}{10^6 C^3}$$

Design For Cyclic Loads and Speeds

- Similarly, the life consumed by the second element is given by $$\frac{N_2 P_2^3}{10^6 C^3}$$
- Adding these expressions, the life consumed by the complete work cycle is given by

$$\frac{N_1 P_1^3}{10^6 C^3} + \frac{N_2 P_2^3}{10^6 C^3} + \cdots + \frac{N_x P_x^3}{10^6 C^3} \qquad \text{(a)}$$

- If *Pe* is the equivalent load for the complete work cycle, the life consumed by the work cycle is given by,

$$\frac{N\, P_e^3}{10^6 C^3} \qquad \text{(b)}$$

- Equating expressions (a) and (b),

$$N_1 P_1^3 + N_2 P_2^3 + \ldots + N_x P_x^3 = N P_e^3$$

Design For Cyclic Loads and Speeds

- Or,

$$P_e = \sqrt[3]{\left[\frac{N_1 P_1^3 + N_2 P_2^3 + \cdots}{N_1 + N_2 + \cdots}\right]}$$

$$P_e = \sqrt[3]{\left[\frac{\sum N P^3}{\sum N}\right]}$$

- This equation is used for calculating the dynamic load capacity of a bearing

Design For Cycling Loads and Speeds

- When the load does not vary in steps of constant magnitude, but varies continuously with time, the above equation is modified and written as

$$P_e = \left[\frac{\int_0^N P^3\, dN}{\int_0^N dN} \right]^{1/3}$$

$$P_e = \left[\frac{1}{N} \int P^3\, dN \right]^{1/3}$$

- In case of bearings, where there is a combined radial and axial load, it should be fi rst converted into equivalent dynamic load before the above computations are carried out.

PROBLEM - 1

- *A single-row deep groove ball bearing has a dynamic load capacity of 40500 N and operates on the following work cycle:*

  (i) radial load of 5000 N at 500 rpm for 25% of the time;

  (ii) radial load of 10000 N at 700 rpm for 50% of the time; and

  (iii) radial load of 7000 N at 400 rpm for the remaining 25% of the time.

  Calculate the expected life of the bearing in hours.

SOLUTION

Given C = 40,500 N

Step I ; Equivalent load for complete work cycle

Consider the work cycle of one minute duration.

The values of load P and revolutions N are tabulated as follows:

Element No.	*P (N)*	*Element time (minute)*	*Speed (rpm)*	*Revolutions N in element time*
1	5000	0.25	500	125
2	10000	0.5	700	350
3	7000	0.25	400	100
Total		1.00		575

$$P_e = \sqrt[3]{\left[\frac{N_1 P_1^3 + N_2 P_2^3 + N_3 P_3^3}{N_1 + N_2 + N_3}\right]}$$

$$= \sqrt[3]{\left[\frac{125(5000)^3 + 350(10\,000)^3 + 100(7000)^3}{575}\right]}$$

$$= 8860.06 \text{ N}$$

- ***Step II ; Bearing life (L_{10h})***

According to the load life relationship,

$$L_{10} = \left(\frac{C}{P_e}\right)^3$$

$$= \left(\frac{40\,500}{8860.06}\right)^3$$

$$= 95.51 \text{ million rev.}$$

$$L_{10h} = \frac{L_{10} \times 10^6}{60n}$$

$$= \frac{95.51 \times 10^6}{60(575)}$$

$$= 2768.45 \text{ h}$$

PROBLEM - 2

- *A ball bearing is operating on a work cycle consisting of three parts—a radial load of 3000 N at 1440 rpm for one quarter cycle, a radial load of 5000 N at 720 rpm for one half cycle, and radial load of 2500 N at 1440 rpm for the remaining cycle. The expected life of the bearing is 10 000 h. Calculate the dynamic load carrying capacity of the bearing.*

Given $L_{10h} = 10\ 000\ \mathrm{h}$

Solution

- ***Step I ; Equivalent load for complete work cycle.***

Considering work cycle of one minute duration,

$$N_1 = \frac{1}{4}(1440) = 360 \text{ rev.}$$

$$N_2 = \frac{1}{2}(720) = 360 \text{ rev.}$$

$$N_3 = \frac{1}{4}(1440) = 360 \text{ rev.}$$

The average speed of rotation is given by

$$n = N_1 + N_2 + N_3$$

$$= 1080 \text{ rpm}$$

$$P_e = \sqrt[3]{\left[\frac{N_1 P_1^3 + N_2 P_2^3 + N_3 P_3^3}{N_1 + N_2 + N_3}\right]}$$

$$= \sqrt[3]{\left[\frac{360(3000)^3 + 360(5000)^3 + 360(2500)^3}{1080}\right]}$$

$$= 3823 \text{ N}$$

Solution

- ***Step II ; Dynamic load carrying capacity of bearing.***

$$L_{10} = \frac{60nL_{10h}}{10^6}$$

$$= \frac{60(1080)(10\,000)}{10^6}$$

$$= 648 \text{ million rev.}$$

Now,

$$C = P(L_{10})^{1/3}$$

$$= 3823(648)^{1/3}$$

$$= 33\,082 \text{ N}$$

BEARING WITH A PROBABILITY OF SURVIVAL OTHER THAN 90 PER CENT

- In the definition of rating life, it is mentioned that the rating life is the life that 90% of a group of identical bearings will complete or exceed before fatigue failure.
- The reliability *R* is defined as

$$R = \frac{\text{No. of bearings which have successfully completed } L \text{ million revolutions}}{\text{Total number of bearings under test}}$$

- Therefore, the reliability of bearings selected from the manufacturer's catalogue is 0.9 or 90%.
- In certain applications, where there is risk to human life, it becomes necessary to select a bearing having a reliability of more than 90%.

Bearing with a Probability of Survival other than 90 Per Cent

- Figure shows the distribution of bearing failures. The relationship between bearing life and reliability is given by a statistical curve known as *Wiebull distribution*.

For Wiebull distribution,

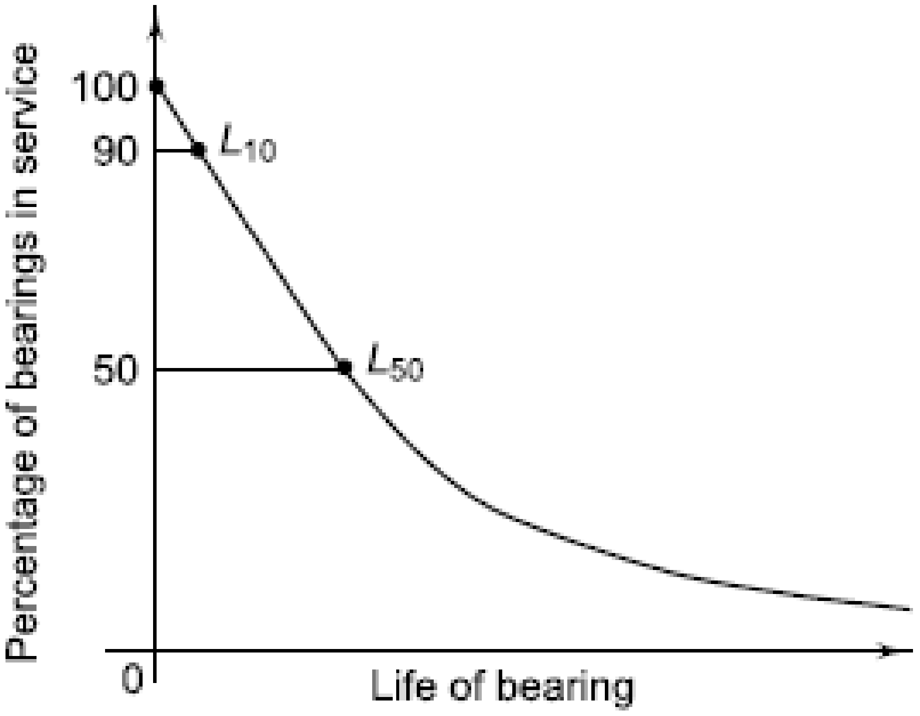
$$R = e^{-(L/a)^b}$$

wnere,

R is the reliability (in fraction),

L is the corresponding life

a & b are constants.



Relationship between bearing life and reliability

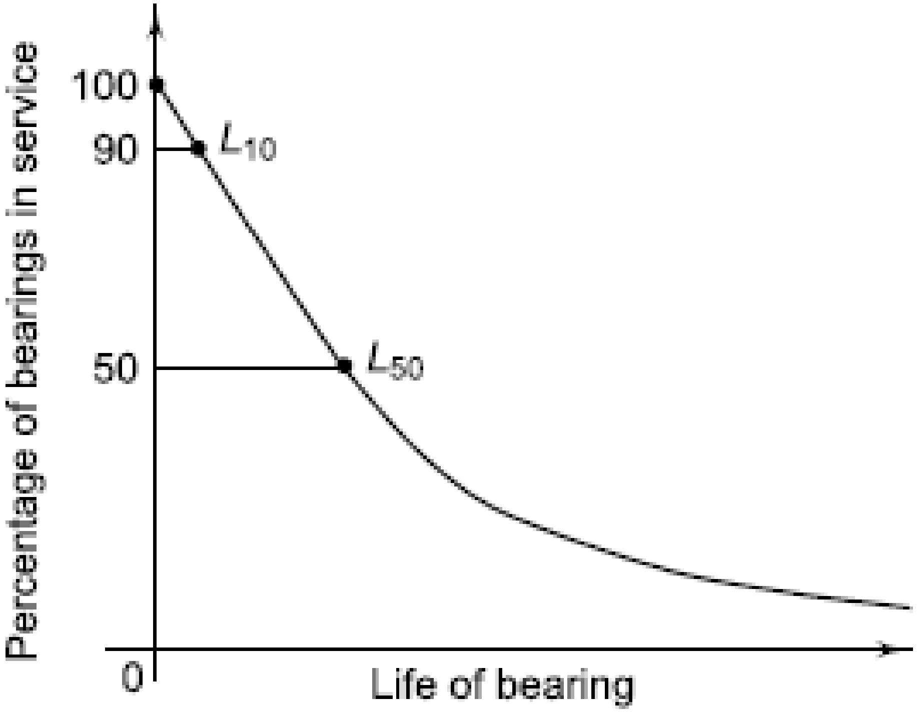
Bearing with a Probability of Survival other than 90 Per Cent

$$R = e^{-(L/a)^b}$$

Rearranging the above equation, we have

$$\left(\frac{1}{R}\right) = e^{(L/a)^b}$$

$$\log_e\left(\frac{1}{R}\right) = \left(\frac{L}{a}\right)^b$$



Wiebull distribution

- If L_{10} is the life corresponding to a reliability of 90% or $R90$.

$$\log_e\left(\frac{1}{R_{90}}\right) = \left(\frac{L_{10}}{a}\right)^b$$

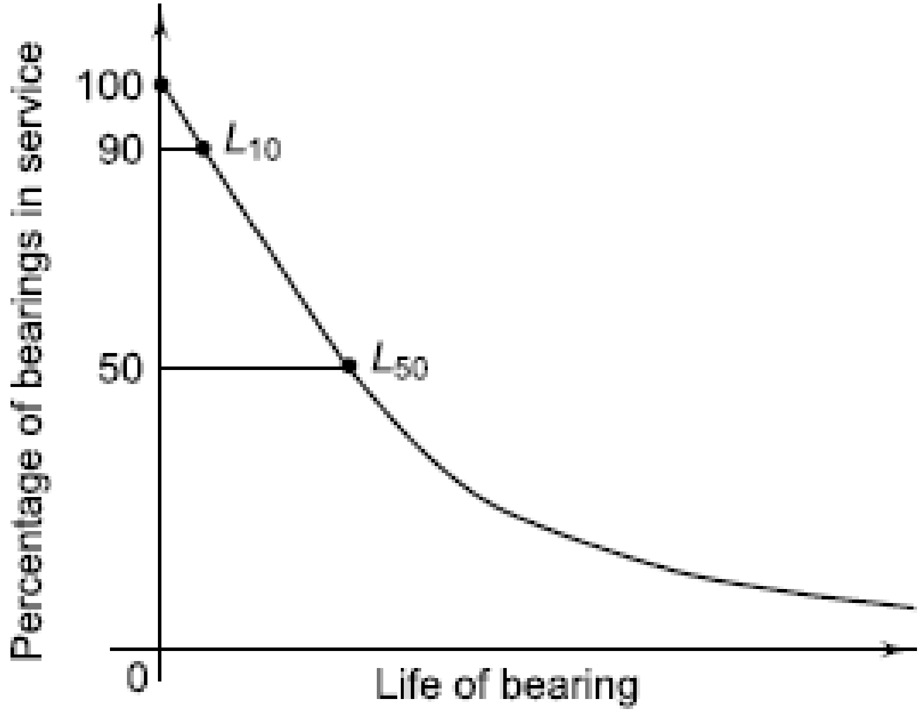
Dividing Eq. (a) by Eq. (b), we hav

$$\left(\frac{L}{L_{10}}\right) = \left[\frac{\log_e\left(\frac{1}{R}\right)}{\log_e\left(\frac{1}{R_{90}}\right)}\right]^{1/b}$$

where $R_{90} = 0.9$

The values of a and b are

$$a = 6.84 \quad \text{and} \quad b = 1.17$$



Problem

A single-row deep groove ball bearing is subjected to a radial force of 8 kN and a thrust force of 3 kN. The values of X and Y factors are 0.56 and 1.5 respectively. The shaft rotates at 1200 rpm. The diameter of the shaft is 75 mm and Bearing No. 6315 (C =112 000 N) is selected for this application. Estimate

(i) the life of this bearing, with 90% reliability.

(ii) Estimate the reliability for 20 000 h life.

Given Data :

Step I ; Bearing life with 90% reliability

$$F_r = 8 \text{ kN}$$

$$F_a = 3 \text{ kN}$$

$$X = 0.56 \quad Y = 1.5$$

$$n = 1200 \text{ rpm}$$

$$d = 75 \text{ mm}$$

$$C = 112\,000 \text{ N}$$

$$P = XF_r + YF_a$$
$$= 0.56\,(8000) + 1.5\,(3000)$$
$$= 8980 \text{ N}$$

$$L_{10} = \left(\frac{C}{P}\right)^3$$
$$= \left(\frac{112\,000}{8980}\right)^3$$
$$= 1940.10 \text{ million rev.}$$

$$L_{10h} = \frac{L_{10}(10^6)}{60n}$$
$$= \frac{1940.10(10^6)}{60(1200)}$$
$$= 26\,945.83 \text{ h}$$

...

- ***Step II ; Reliability for 20 000 hr life***

$$\left(\frac{L}{L_{10}}\right)=\left[\frac{\log_e\left(\frac{1}{R}\right)}{\log_e\left(\frac{1}{R_{90}}\right)}\right]^{1/b}$$

$$\left(\frac{L}{L_{10}}\right)^b=\left[\frac{\log_e\left(\frac{1}{R}\right)}{\log_e\left(\frac{1}{R_{90}}\right)}\right]$$

Substituting the following values,

$$L = 20\,000 \text{ h}$$

$$L_{10} = 26\,945.83 \text{ h}$$

$$R_{90} = 0.90$$

$$b = 1.17$$

- We get

$$\left(\frac{20\,000}{26\,945.83}\right)^{1.17}=\left[\frac{\log_e\left(\frac{1}{R}\right)}{\log_e\left(\frac{1}{0.90}\right)}\right]$$

$$\therefore \quad R = 0.9283 \text{ or } 92.83\%$$

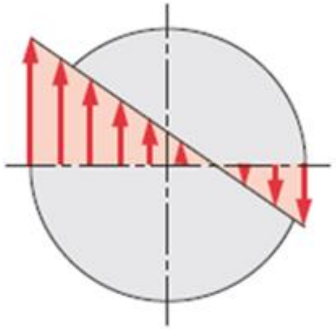
- ***End of Lecture***

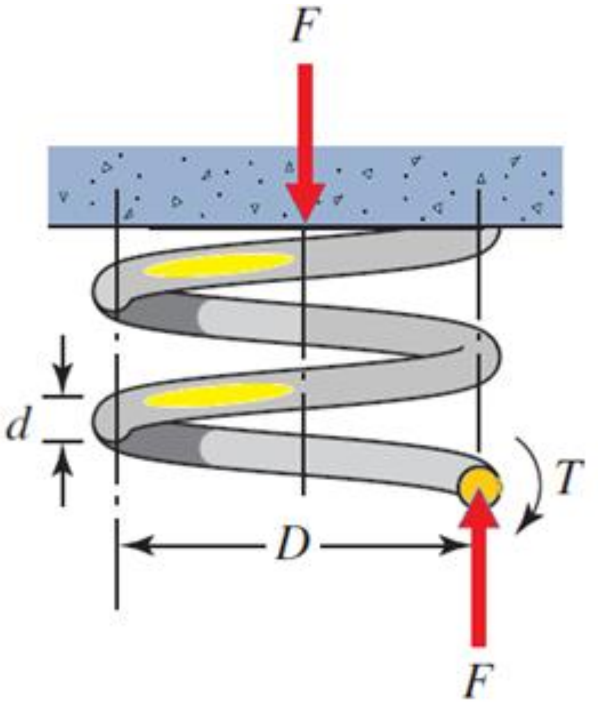
MACHINE DESIGN II
MEC 3110

Lecture - 15
Mechanical Springs
(1)

By
PROF. M. NAUSHAD ALAM

MECHANICAL ENGINEERING DEPT.
A.M.U. ALIGARH





Springs Defined

A resilient machine element,

which deflects under the action of the load

and returns to its original shape when the load is removed.

- A spring is capable of providing large elastic deformation.
- Springs in general, are of following types:
    - Coil springs
    - Flat springs
    - Special-shape springs

Functions of springs

1. To store energy:

   Springs absorb shocks and vibrations by storing energy.

2. To measure forces.

   Springs used in w

3. To apply forces

   Springs maintain contact between the two machine elements by applying forces.

4. To control the motion

   of machine elements.

Applications of springs

1. Springs are used for applying forces have the following applications:

   In the cam and follower mechanism; maintain contact between the machine elements

   In engine valve mechanism; return the rocker arm to its normal position

   In clutches, springs provides the force required for engagement and disengagement

4. Springs used for controlling the motion have the following applications

   in clocks,

   toys,

   movie-cameras,

   circuit breakers and starters.

Applications of springs

1. Springs are used to absorb shocks and vibrations have following applications

   vehicle suspensions,

   railway buffer springs,

   buffer springs in elevators and

   vibration mounts for machines

2. Springs are also used to measure forces and have the following applications

   in weighing balances

   in measuring scales

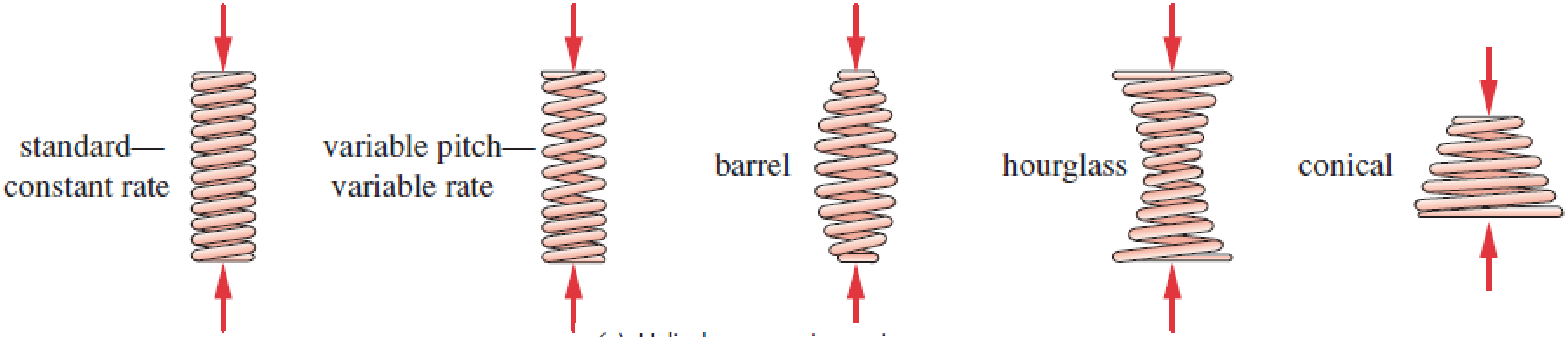
SPRING CONFIGURATIONS

Springs can have many shapes, and can be categorized according to their physical configurations as :

1. Wire-form springs
2. Flat springs
3. Spring washers
4. Flat-wound springs

1. **Wire-form springs**

helical compression springs

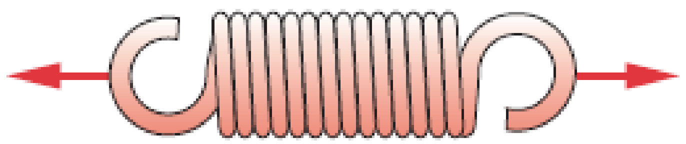


(a) Helical compression springs.

Spring Configurations

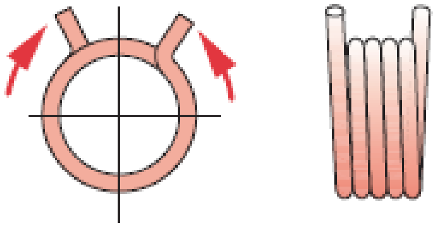
1. **Wire-form springs**

- **helical tension,**



(b) Helical extension springs.

- **helical torsion**



(d) Torsion springs

Spring Configurations

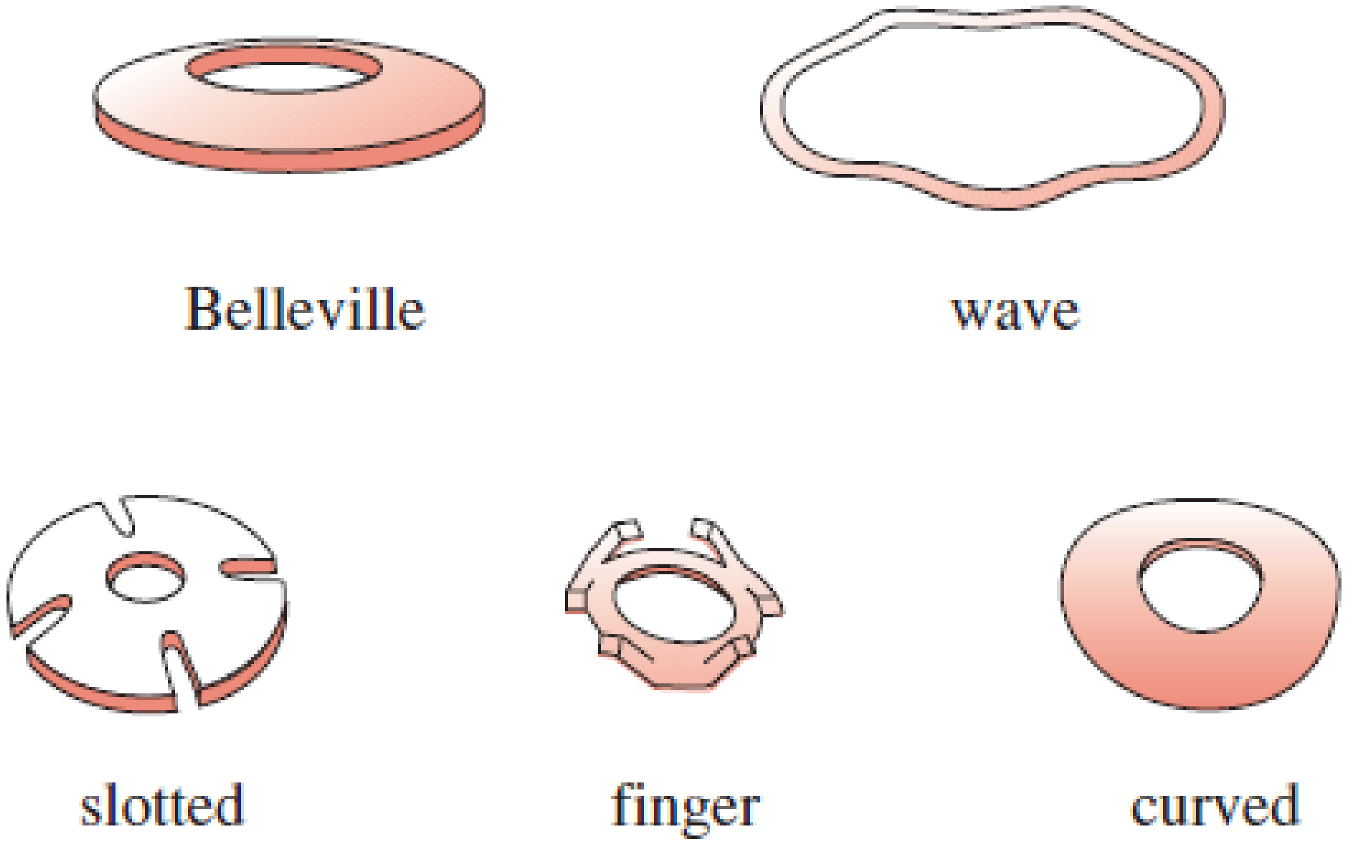
3. Spring washers

curved,

wave,

finger,

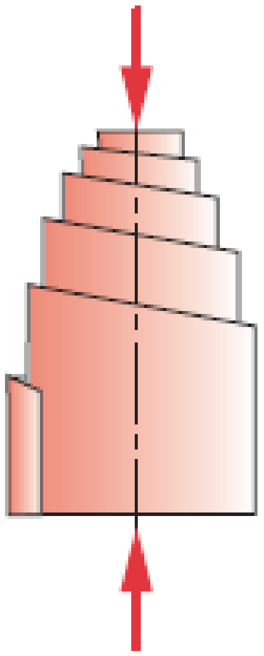
Belleville.



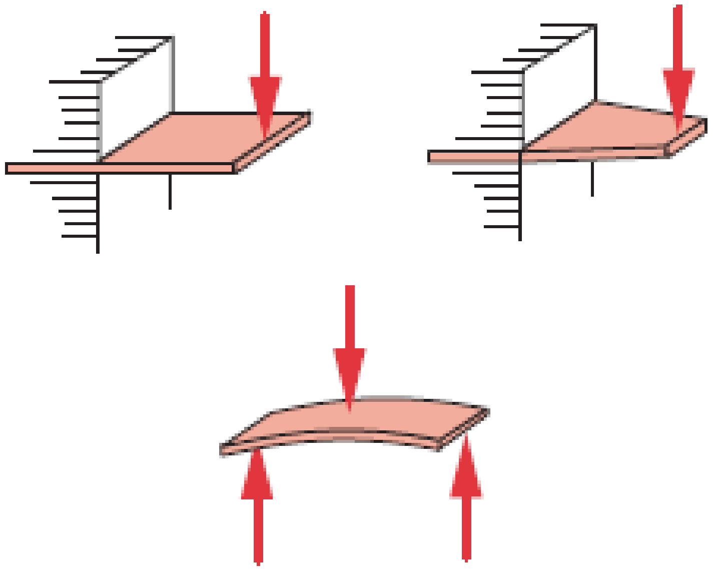
Spring Configurations

2. **Flat springs**

Varieties of **flat springs**.



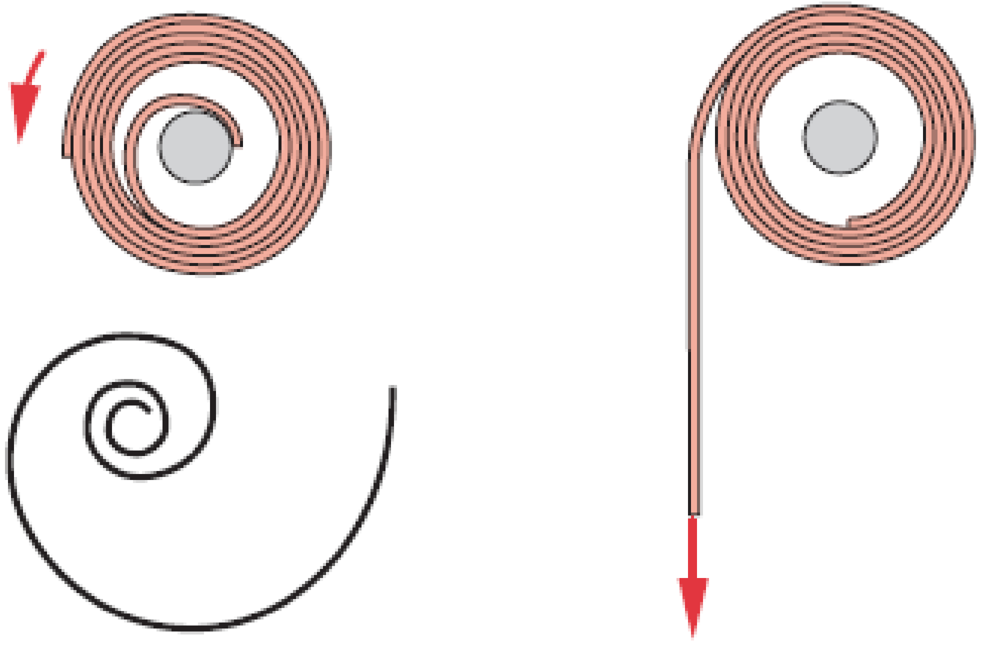
Volute spring.



Beam springs.

Spring Configurations

4. Flat-wound springs

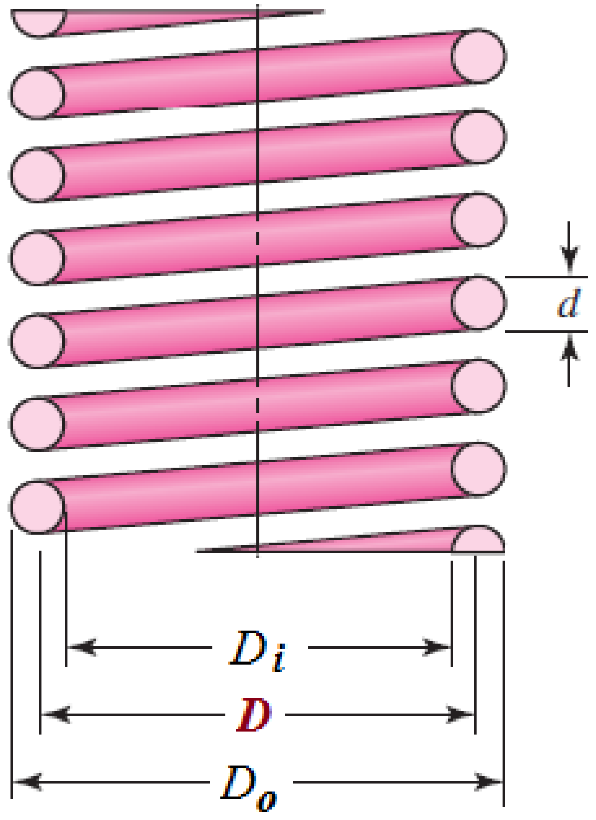


Power or motor springs.

Constant Force.

HELICAL COMPRESSION SPRINGS

These springs are the most widely used ones.



Terminology

d = wire diameter of spring (mm)

Di = inside diameter of spring coil (mm)

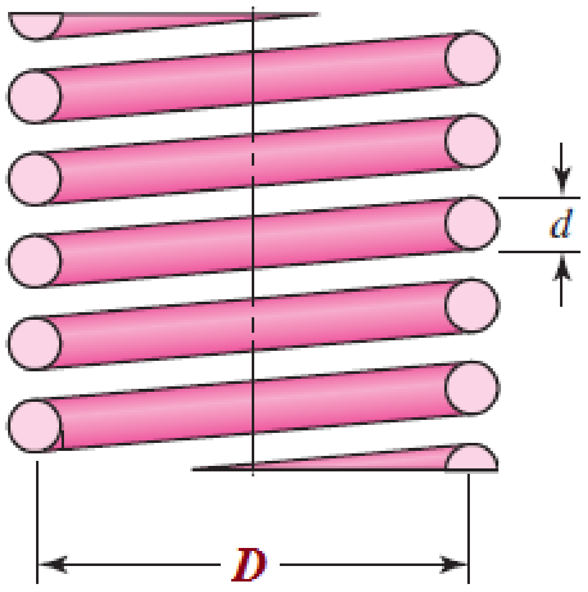
Do = outside diameter of spring coil (mm)

D = mean coil diameter (mm)

SPRING INDEX

- *The spring index is* an important parameter. It is *defined as the ratio of mean coil diameter to wire diameter*. Denoted by the letter *C*.

$$C = \frac{D}{d}$$



- It indicates the relative sharpness of the curvature of the coil.
- The designer should have a good judgement in selecting the value of C. Low spring index means high sharpness of curvature.

Low spring index; **C < 3,**

- The actual stresses in the wire are excessive due to curvature effect.
- Such a spring is difficult to manufacture; Special care is required in coiling to avoid cracking of the wire.

spring index

High spring index; C > 15

- This results in large variations of the coil diameter.
- As such the spring is prone to buckling.

Practical spring index

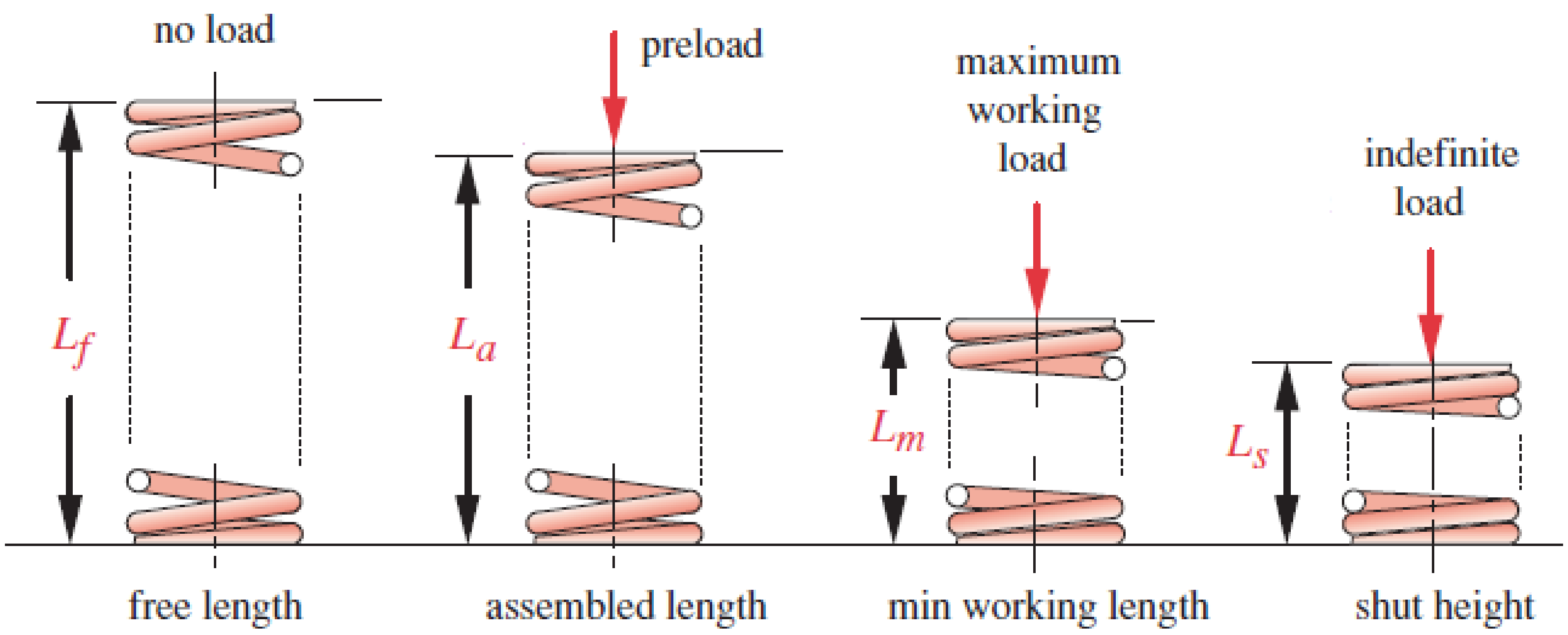
C = 4 to 12

- This range is the best from the point of view of manufacturing considerations.
- A spring index in the range of 6 to 9 is particularly suited for close tolerance springs and springs subjected to cyclic loads.

TERMS RELATED TO HELICAL SPRINGS

There are four important terms related to spring configurations:

1. *Free length*
2. *Assembled length*
3. *Minimum working length*
4. *Shut height*



Terms Related to Helical Springs

1. Free length; L_f

- ***The overall spring length in the unloaded condition, i.e., as manufactured***..
- In this case, no external force acts on the spring.
- Free length is the length of the spring in free condition prior to assembly

2. Assembled Length; L_a

- ***It is the length of the spring after installation to its initial deflection $y_{initial}$.***
- *This initial deflection in combination with spring rate k determines preload force required at assembly.*

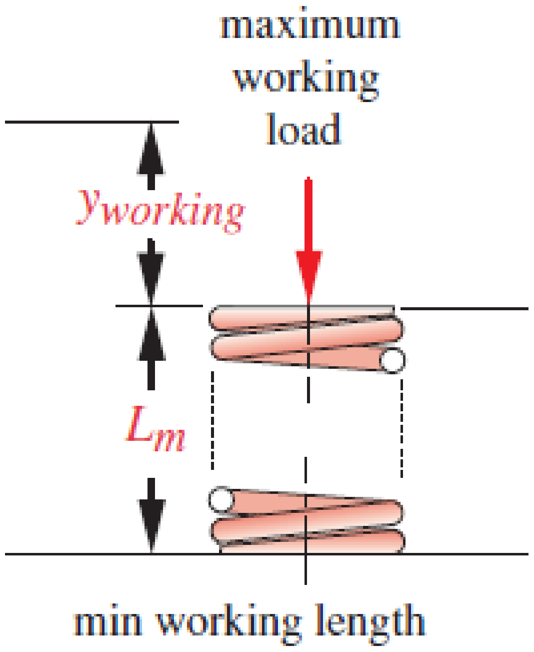
.

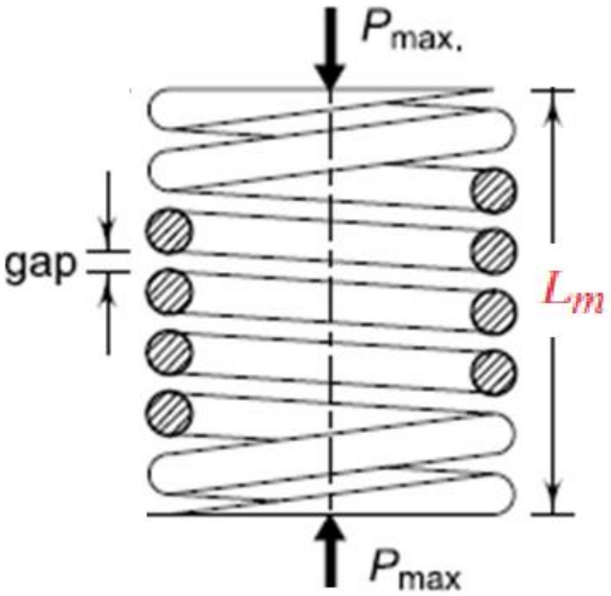
Terms Related to Helical Springs

3. Minimum Working Length; L_m

- The working load on the spring further compresses it to its working deflection $y_{working}$.
- ***The shortest length of the spring to which it is compressed in service.***
- Under this condition, there should be some gap or clearance between the adjacent coils. The gap is essential to prevent clashing of the coils.

- *Sometimes, an arbitrary decision is taken and it is assumed that there is a gap of 1 or 2 mm between adjacent coils under maximum load condition.*





Terms Related to Helical Springs

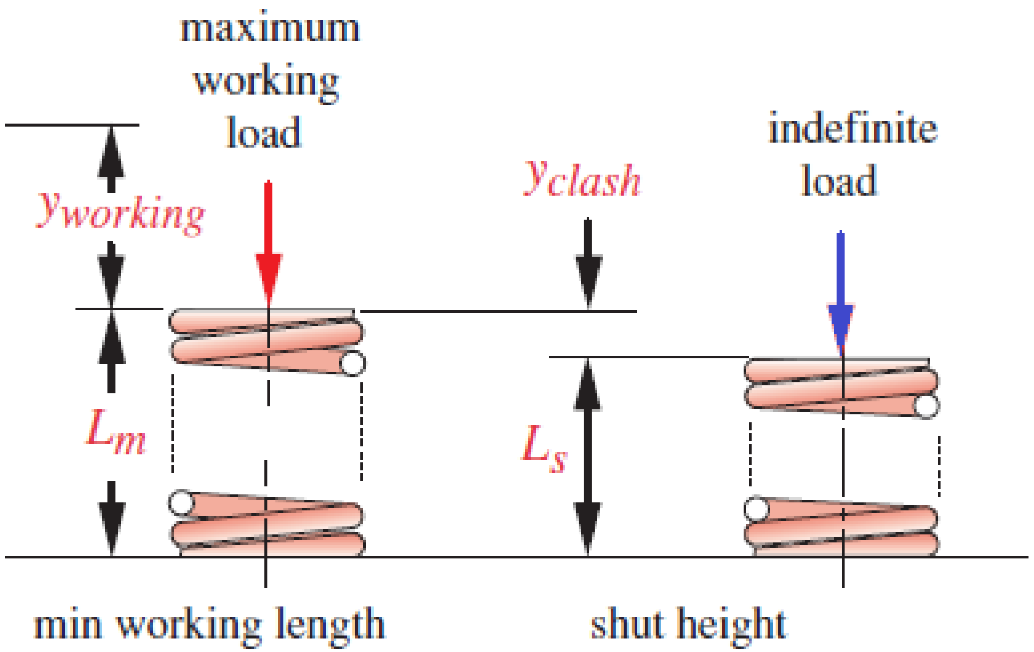
4. *Shut height or solid height* L_s

- The length to which spring gets compressed when all coils are in contact. Once shut, the spring can support “indefinite” loads up to the compressive strength of wire.

- The clash allowance y_{clash} is the difference between the minimum working length and the shut height.

  It is expressed as a percentage of the working deflection.

- A minimum clash allowance of 10–15% is recommended to avoid reaching the shut height in service.



Expressions for spring Lengths

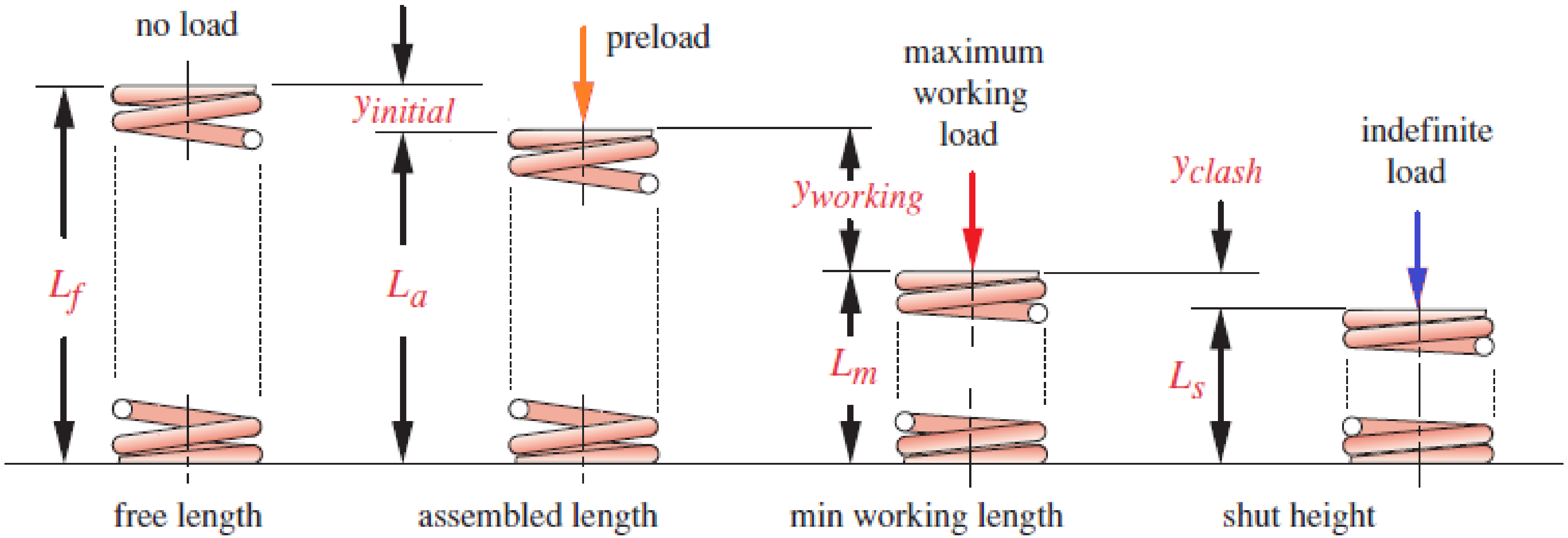
- Solid length = $N\boldsymbol{t}.d$,

  where, $N\boldsymbol{t}$ = total number of coils and d is the diameter of spring wire.

- Free length = Assembled length + $y_{initial}$

  = solid length + $y_{working}$ + + $y_{working}$ + y_{clash}

+ δ

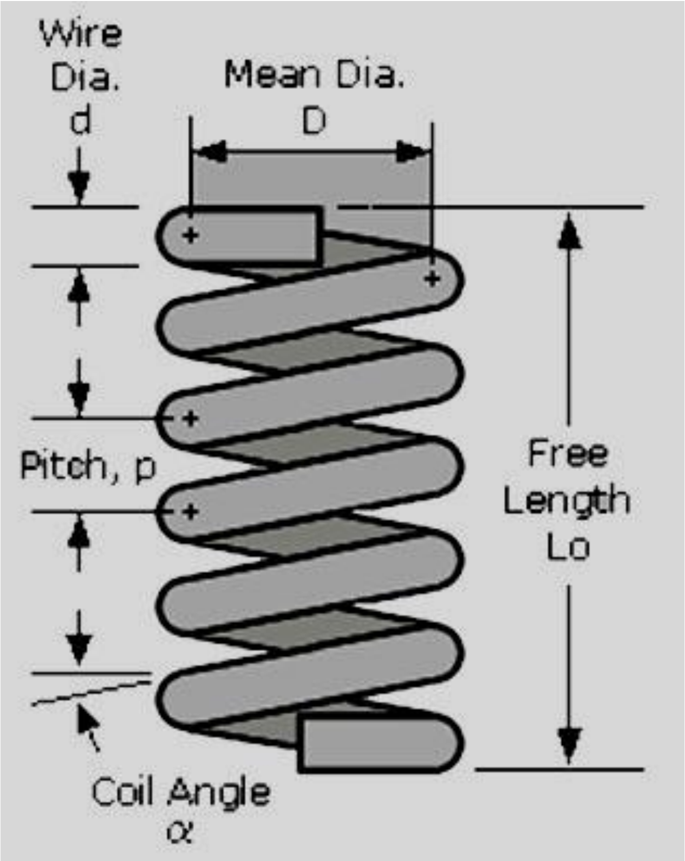


Pitch of the Coil

- The axial distance between adjacent coils in uncompressed state of spring.

  It is denoted by ***p***

  $p = L_f / (Nt - 1)$



Stiffness of the spring

Defined as the force required to produce unit deflection in the spring.

Stiffness $k = \dfrac{P}{\delta}$ (N/mm)

where, *P* = axial spring force (N)

d = axial deflection of the spring.

The stiffness of spring is also known by the following names:

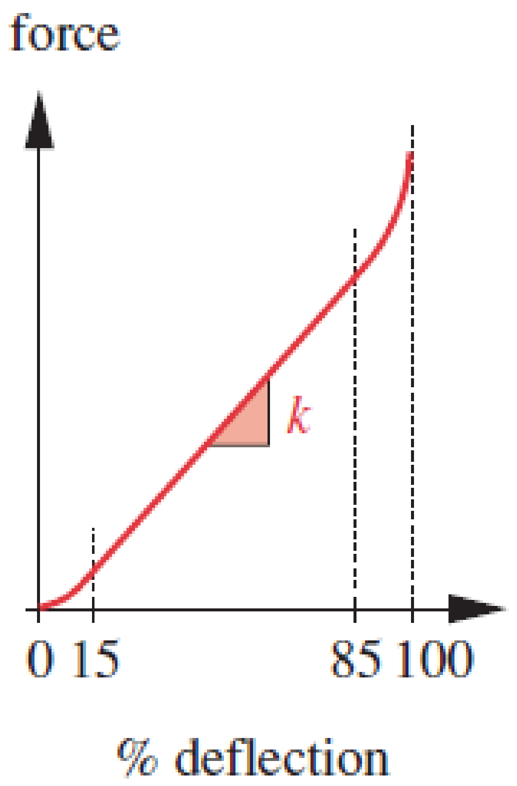
rate of spring,
gradient of spring,
scale of spring
spring constant.

Stiffness of the spring

- The spring stiffness represents the slope of the load-deflection curve for the spring.

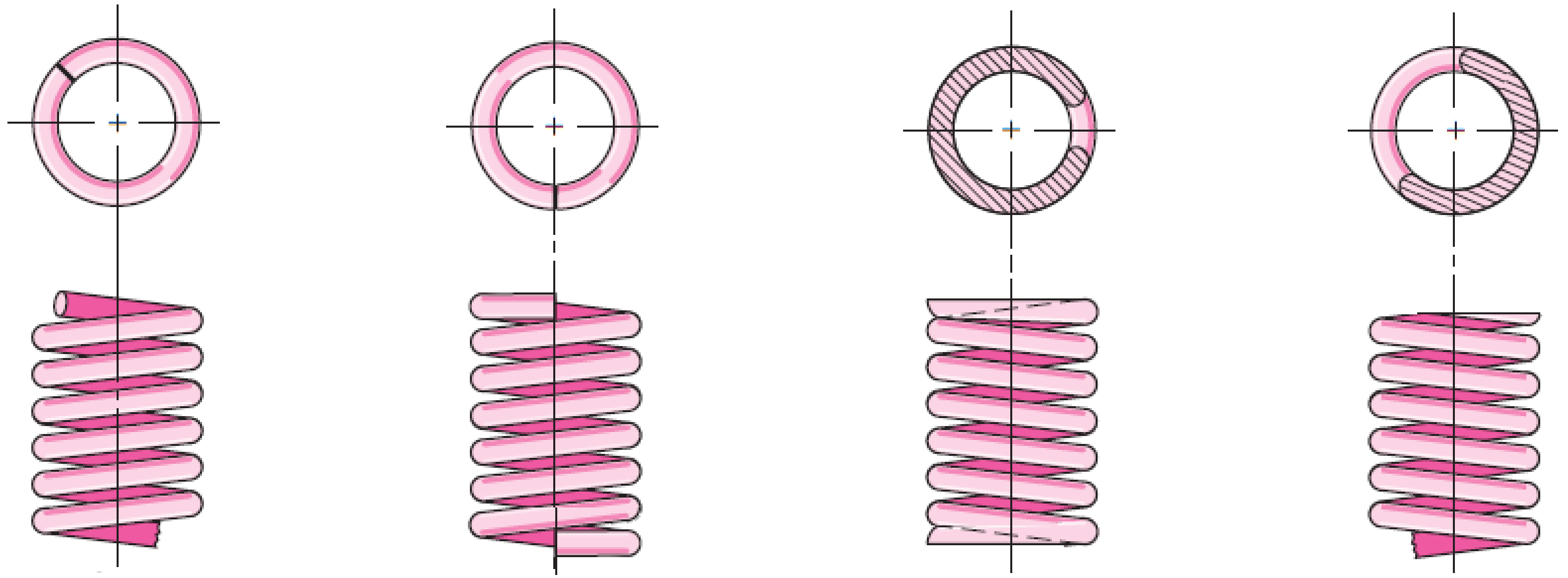
- The spring rate for a constant-pitch helical compression spring is linear over most of its operating range.

- Only the first and last few percent of its deflection is nonlinear.



Active Coils of a Spring

- All the coils in a spring may or may not contribute actively to the spring's deflection.
- The active coils are the ones which deflect under the action of applied forces forces.
- The no. of active coils N_a present in a spring, depends on the end treatment.
- There are four end-coil conditions



$N_a = N_t$ Plain end

$N_a = N_t - 1$ (*b*) Squared or closed end,

$N_a = N_t - 2$ (*c*) Squared and ground end,

(*d*) Plain end, ground,

…

- Squared ends effectively remove two coils from active deflection.
- Grinding by itself removes 1 active coil.
- The calculated number of active coils is usually rounded to the nearest 1/4 coil, as the manufacturing process cannot always achieve better than that accuracy.

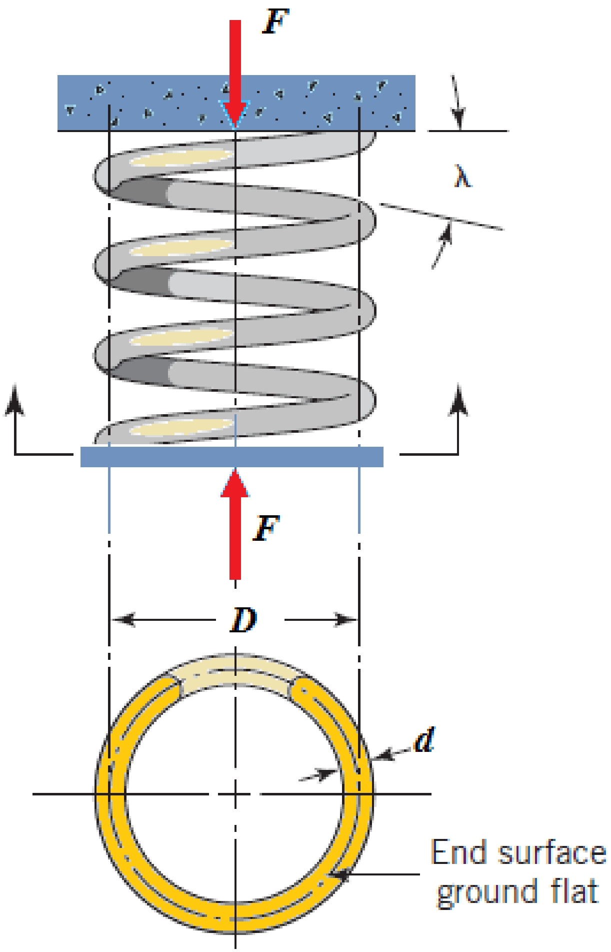
.

STRESSES IN HELICAL SPRINGS

- An external force F is assumed to be applied *along the axis of the helix*.
- This is achieved by winding the end coils with zero pitch and then grinding the ends flat.
- In this way the pressure applied by the end plates is uniformly distributed.
- We take a horizontal cutting plane and draw the FBD of a portion of the spring.
- The wire is subjected to

  (1) a transverse shear force of F

  (2) a torque equal to $FD/2$.
- The entire length of active wire in the helix is subjected to torque.



Stresses In Helical Springs

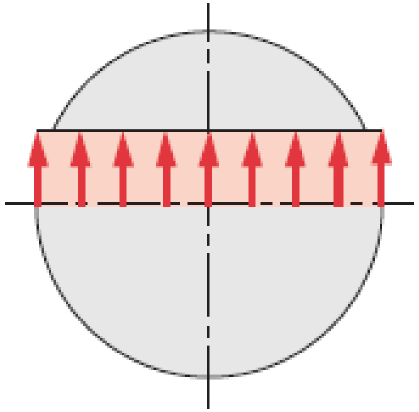
- For a round solid wire coil spring, the direct stress is

$$\tau_d = \frac{4F}{\pi d^2}$$

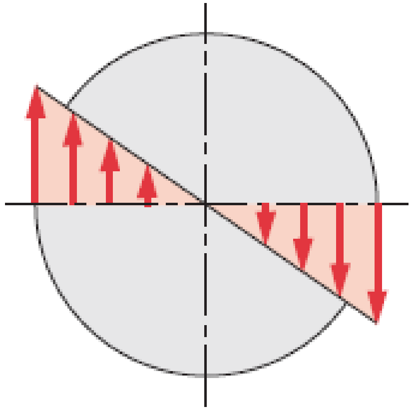
- For a round solid wire coil spring, the resulting torsional stress is

$$\tau = \frac{Tr}{J}$$

$$= \frac{16T}{\pi d^3} = \frac{8FD}{\pi d^3}$$



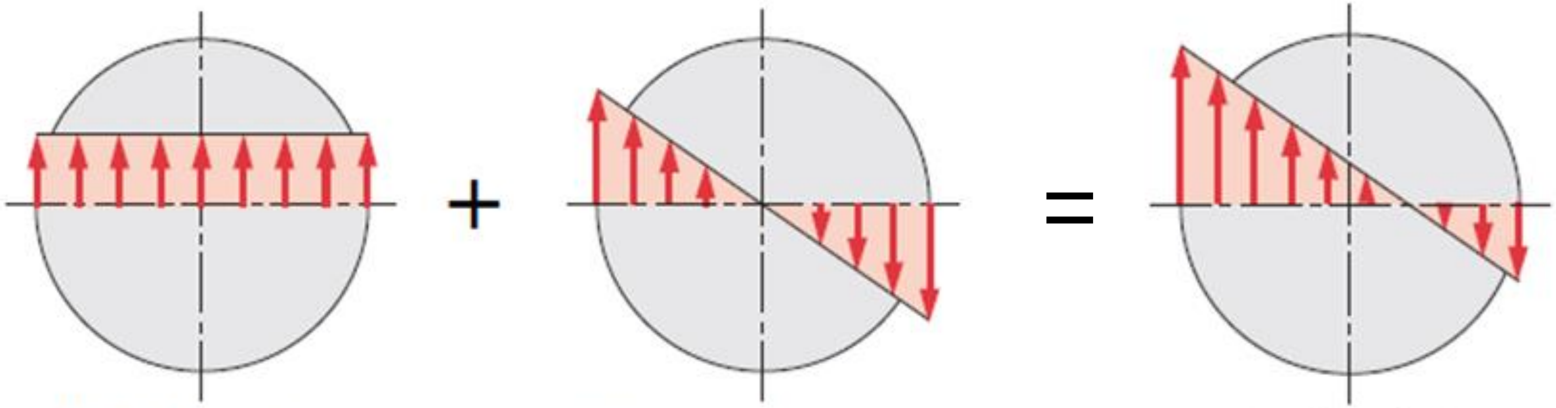
Direct shear stress



Torsional shear stress

Stresses In Helical Springs

- The two shear stresses add directly,



Direct shear stress + Torsional shear stress = Combined stress

- Maximum shear stress occurs at the inner fibre of the wire's cross section,

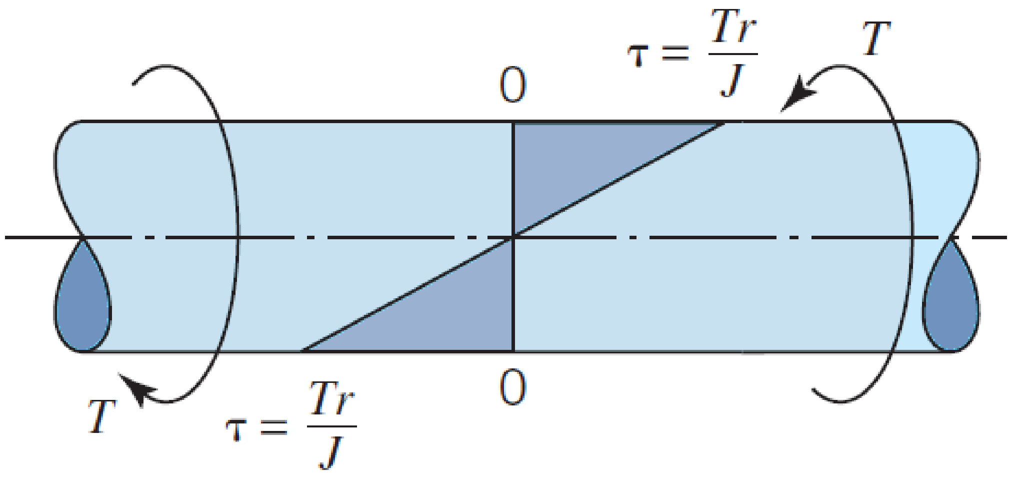
$$\tau_{max} = \tau_{t} + \tau_{d}$$

$$= \frac{8FD}{\pi d^3} + \frac{4F}{\pi d^2}$$

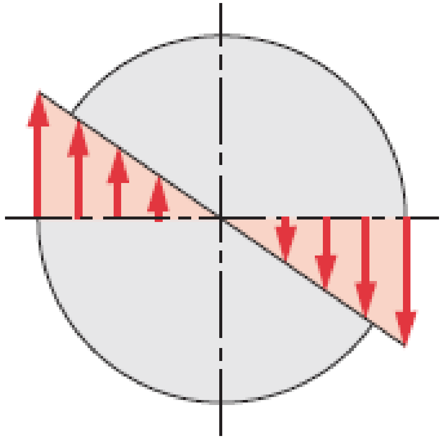
Stresses In Helical Springs

- Torsional shear stress is $$\tau = \frac{8FD}{\pi d^3}$$

- Thus, a helical compression or tension spring can be thought of as a torsion bar wound in the form of a helix.



Straight torsion bar



Torsional shear stress

Stresses In Helical Springs

- The maximum shear stress occurs at the inner fiber of the wire's cross section,

$$\tau_{max} = \tau_{t} + \tau_{d} = \frac{8FD}{\pi d^3} + \frac{4F}{\pi d^2}$$

$$= \frac{8FD}{\pi d^3}\left(1 + \frac{0.5}{C}\right)$$

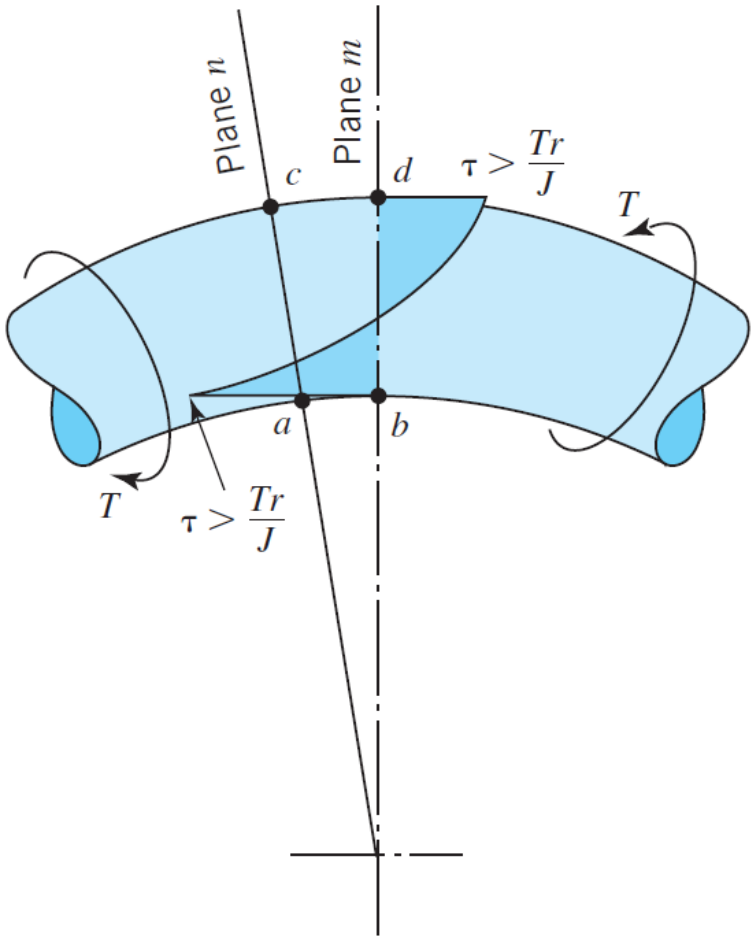
$$= K_s \frac{8FD}{\pi d^3}$$

The factor K_s is a **direct shear stress factor.**

Stresses In Helical Springs

Curvature Effect:

- In addition, the inner surfaces of the coil is subjected to an increase in the intensity of torsional stress due to its curvature.
- This effect is similar to stress concentration due to shifting of the neutral axis away from the geometric center in curved beams.
- This makes the already high stress at the inner surface of the coil even higher



Curved torsion bar

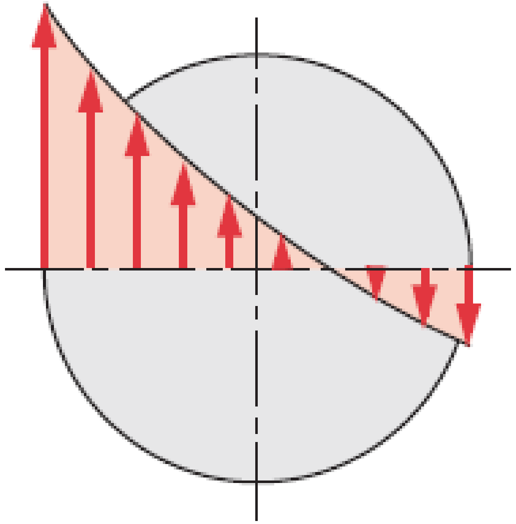
Stresses In Helical Springs

- Wahl determined the stress-concentration factor for round wires and defined a factor *Kw*.

$$K_w = \frac{4C-1}{4C-4} + \frac{0.615}{C}$$

- The Wahl's factor incudes both the direct shear effects and the stress concentration for round wires with $C \geq 1.2$

- This effect can be neglected for static loading, because local yielding with the first application of the load will relieve it.



Effects of stress concentration at inside edge

Deflection and Stiffness of Helical spring

- The deflection-force relations can be obtained -- using ***Castigliano's theorem.***
- The total strain energy of spring is composed of two components

1. Torsional component

$$U_{Torsion} = \frac{F^2 l}{2AE}$$

2. Shear component.

$$U_{Shear} = \frac{T^2 l}{2GJ}$$

- Therefore the strain energy of spring

$$U = \frac{T^2 l}{2GJ} + \frac{F^2 l}{2AG}$$

- Substituting the following relations:

$$T = FD/2$$

$$l = \pi D N$$

$$J = \frac{\pi d^4}{32}$$

$$A = \frac{\pi d^2}{4}$$

- We get, the total strain energy as;

$$U = \frac{4F^2 D^3 N}{d^4 G} + \frac{2F^2 DN}{d^2 G}$$

- where *N* = *Na are the* number of active coils.

- Now using Castigliano's Theorem the total deflection in the spring, may be written as:

$$y = \frac{\partial U}{\partial F}$$

$$= \frac{\partial}{\partial F}\left(\frac{4F^2D^3N}{d^4G} + \frac{2F^2DN}{d^2G}\right)$$

$$= \frac{8FD^3N}{d^4G} + \frac{4FDN}{d^2G}$$

- Since $C = D/d$, we can write;

$$y = \frac{8FD^3N}{d^4G}\left(1 + \frac{1}{2C^2}\right)$$

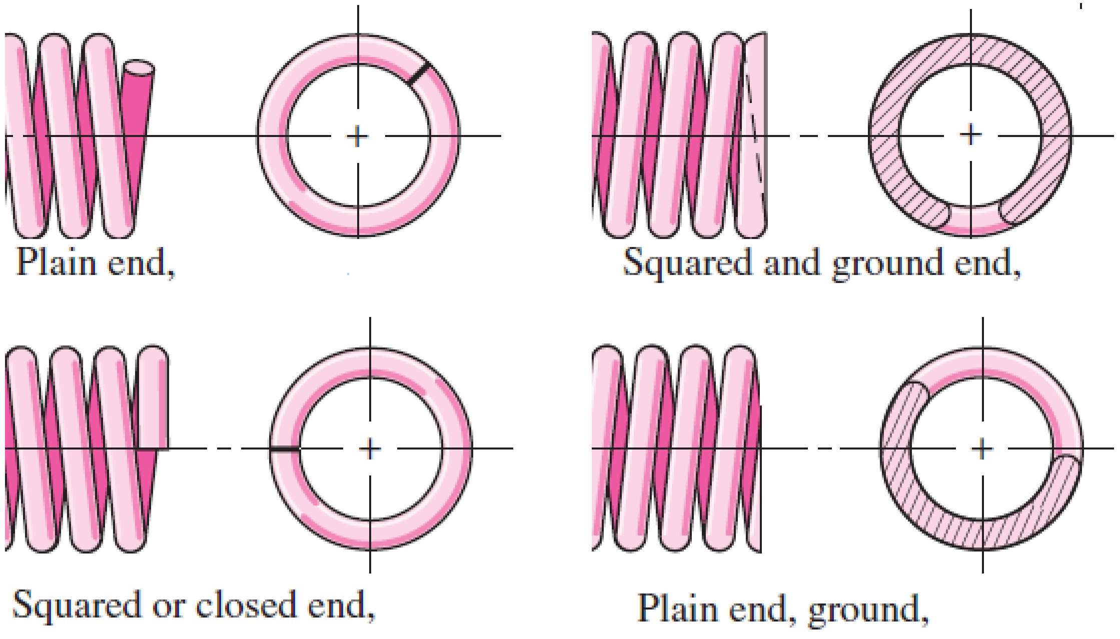
$$\doteq \frac{8FD^3N}{d^4G}$$

- The spring rate or scale of the spring

$$k = \frac{F}{y}$$

$$\doteq \frac{d^4G}{8D^3N}$$

- **T**he spring stiffness, springs rate or the spring scale ; $k = \frac{F}{y} = \frac{d^4 G}{8 D^3 N}$
- Using the equation the number of active coils needed to maintain the desired deflection or spring stiffness can be determined.
- In order to maintain proper contact and align the force along the spring axis the ends are to be properly shaped. The four types of ends generally used for compression springs are:



SET REMOVAL or PRESETTING

- This is a process used in the manufacture of compression springs to induce useful residual stresses.
- It is done by making the spring longer than needed and compressing it to its solid height.
- This operation sets the spring to the required final length.
- Since the torsional yield strength has been exceeded, residual stresses are induced in the spring.
- The direction of the residual stresses are opposite to those induced during the service.
- Thus, the process increases the strength of the springs.
- This is especially useful when the spring is used for energy storage purposes.
- However, this should not be used for springs subjected to fatigue loading

SPRING MATERIALS

Selection of the manufacturing process

Springs are manufactured by two processes:

1. Hot-working processes
2. Cold-working processes

- Selection of the manufacturing process depends upon the size, the spring index, and the properties desired of the spring.
- In general, pre-hardened wire should not be used…. if $D/d < 4$, or if $d > 1/4$ inch.
- Winding of the spring induces residual stresses through bending.
- But these are normal to the direction of the torsional working stresses in a coil spring.
- Quite frequently in spring manufacture, these stresses are relieved after winding, by a **mild thermal treatment.**

Spring Materials

Selection of material

A great variety of spring materials are available to the designer, these include

- Steels:
  - Plain carbon steels,
  - Alloy steels
  - Corrosion-resisting steels
- Nonferrous materials such as
  - phosphor bronze,
  - spring brass
  - beryllium copper
  - Nickel alloys.

Factors influencing selection of material

The selection of the wire material depends upon the following factors:

(i) The load acting on the spring

(ii) The range of stress through which the spring operates

(iii) The limitations on mass and volume of spring

(iv) The expected fatigue life

(v) The environmental conditions in which the spring will operate such as temperature and corrosive atmosphere.

(vi) The severity of deformation encountered while making the spring.

Spring Materials

Spring Steels

- There are four basic varieties of steel wires which are used in springs :

  (i) patented and cold-drawn steel wires (unalloyed)

  (ii) oil-hardened and tempered spring steel wires and valve spring wires

  (iii) oil-hardened and tempered steel wires (alloyed)

  (iv) stainless steel spring wires

- The most extensively used spring material is high-carbon hard-drawn spring steel.

  It is often called 'patented and cold-drawn' steel wire.

Spring Materials

Spring Steels

Patenting :

- This is the process of heating the steel to above the critical range followed by rapid
- cooling to transform at an elevated temperature from 455° to 465°C.
- This operation produces a tough uniform structure suitable for severe cold drawing.
- After this operation, the spring wire is produced from hot rolled rods by cold drawing through carbide dies to obtain the required diameter.
- The patented and cold drawn steel wires are made of high carbon steel and contain 0.85–0.95% carbon. It

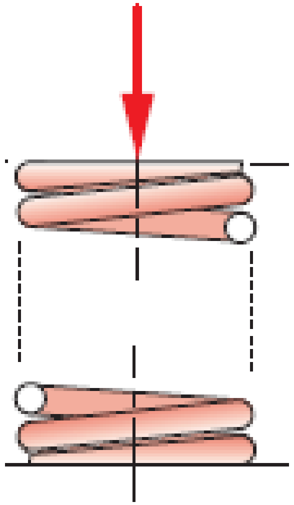
End of Part - 1

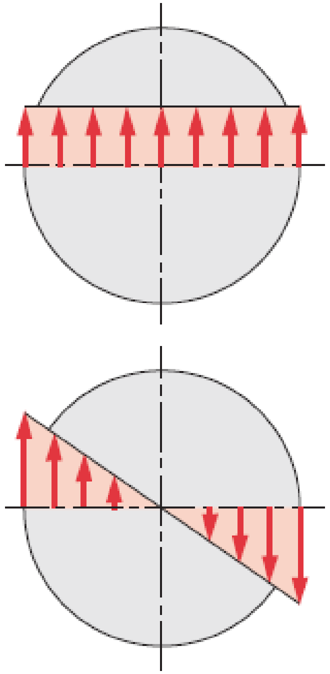
MACHINE DESIGN II
MEC 3110

1

Lecture - 17
DESIGN OF COIL SPRINGS (2)

By
PROF. M. NAUSHAD ALAM





MECHANICAL ENGINEERING DEPT.
A.M.U. ALIGARH

HELICAL COMPRESSION SPRING DESIGN

Terminology

d = wire diameter of spring (mm)

Di = inside diameter of spring coil (mm)

Do = outside diameter of spring coil (mm)

D = mean coil diameter (mm)

C = The spring index

p = Pitch of coils

L_f = Free length

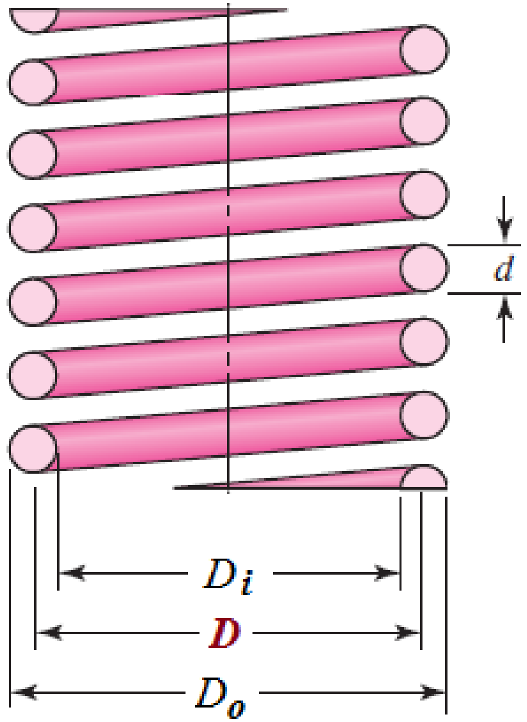
L_a = Assembled length

L_m = Minimum working length

L_s = Shut height

$$C = \frac{D}{d}$$

$$p = L_f \times (N_t - 1)$$



Helical Compression Spring Design

Nomenclature :

A	Material constant
C	Spring index=D/d
d	Wire diameter
D	Mean coil diameter
f	Natural frequency of the spring
F	Force/Load
G	Shear Modulus (of Rigidity)
J	Polar Moment of Inertia
k	Spring rate or spring stiffness
K	Stress correction factor
L	Length
N	Number of coils
T	Torsional Moment
U	Strain energy
	Helix angle
y	Deflection
γ	Density
τ	Shear stress in spring

Design of Compression Coil Springs

Design Consideration :

- The design of a new spring involves the following considerations:-Space into which the spring must fit and operate. -Values of working forces and deflections. -Accuracy and reliability needed

- The primary consideration in the design of the coil springs are that the induced stresses are below the permissible limits while subjected to or exerting the external force F capable of providing the needed deflection or maintaining the spring rate desired.

DESIGN OF HELICAL SPRINGS

There are three objectives for the design of the helical spring. They are as follows:

(i) It should possess suffi cient strength to withstand the external load.

(ii) It should have the required load-deflection characteristic.

(iii) It should not buckle under the external load.

Factor of Safety

The factor of safety in the design of springs is usually 1.5 or less. The use of a relatively low factor of safety is justified on the following grounds:

(i) In most of the applications, springs operate with well defined deflections. Therefore, the forces acting on the spring and corresponding stresses can be precisely calculated.

It is not necessary to take higher factor of safety to account for uncertainty in external forces acting on the spring.

(ii) In case of helical compression springs, an overload will simply close up the gaps between coils without a dangerous increase in deflection and stresses.

(iii) In case of helical extension springs, usually overload stops are provided to prevent excessive deflection and stresses.

(iv) The spring material is carefully controlled at all stages of Manufacturing. The thin and uniform wire cross-section permits uniform heat treatment and cold working of the entire spring.

Therefore, the factor of safety based on torsional yield strength (*Ssy*) is taken as 1.5 for the springs that are subjected to static force.

$$\tau = \frac{S_{sy}}{1.5}$$

Assuming, $S_{yt} = 0.75 S_{ut}$ and $S_{sy} = 0.577 S_{yt}$

Expression (a) is written as,

$$\tau = \frac{(0.577)(0.75) S_{ut}}{1.5}$$

or $$\tau \cong 0.3\ S_{ut} \qquad ($$

…

- The permissible shear stress is, therefore, 30% of the ultimate tensile strength of the spring wire.
- The Indian Standard 4454–1981 has recommended a much higher value for the permissible shear stress.
- According to this standard,

$$\boldsymbol{\sigma = S_{ut}}$$

- This is due to higher tensile yield strengths exhibited by the spring wires.
- In design of helical springs, the permissible shear stress (*t*) is taken from 30% to 50% of the ultimate tensile strength (*Sut*).

...

The basic procedure for the design of helical spring consists of the following steps:

(i) For the given application, estimate the maximum spring force (*P*) and the corresponding required deflection (*d*) of the spring.

In some cases, maximum spring force (*P*) and stiffness *k*, which is (*P*/*d*), are specified.

(ii) Select a suitable spring material and find out ultimate tensile strength (*Sut*) from the data.

Calculate the permissible shear stress for the spring wire by following relationship:

$$t = 0.30\ Sut \text{ or } 0.50\ Sut$$

(iii) Assume a suitable value for the spring index (*C*).

For industrial applications, the spring index varies from 8 to 10.

A spring index of 8 is considered as a good value. The spring index for springs in valves and clutches is 5. The spring index should never be less than 3.

(iv) Calculate the Wahl factor by the following equation:

$$K = \frac{4C-1}{4C-4} + \frac{0.615}{C}$$

(v) Determine wire diameter (d) by Eq. (10.13).

$$\tau = K\left(\frac{8PC}{\pi d^2}\right)$$

(vi) Determine mean coil diameter (D) by the following relationship:

$$D = Cd$$

(vii) Determine the number of active coils (N) by Eq. (10.8).

$$\delta = \frac{8PD^3N}{Gd^4}$$

(viii) Decide the style of ends for the spring depending upon the configuration of the application.

Determine the number of inactive coils. Adding active and inactive coils, find out the total number of coils (*Nt*).

(ix) Determine the solid length of the spring by the following relationship:

Solid length = *Ntd*

(x) Determine the actual deflection of the spring by Equ*ation*

$$\delta = \frac{8PD^3N}{Gd^4}$$

(xi) Assume a gap of 0.5 to 2 mm between adjacent coils, when the spring is under the action of maximum load. The total axial gap between coils is given by,

total gap = (*Nt* – 1) \ gap between two adjacent coils

In some cases, the total axial gap is taken as 15% of the maximum deflection:

....

(xii) Determine the free length of the spring by the following relationship:

free length = solid length + total gap + *d*

(xiii) Determine the pitch of the coil by the following relationship:

$$p = \frac{\text{free length}}{(N_t - 1)}$$

(xiv) Determine the rate of spring by Eq. (10.9).

$$k = \frac{Gd^4}{8D^3N}$$

(xv) Prepare a list of spring specifications.

Problem

It is required to design a helical compression spring subjected to a maximum force of 1250 N. The deflection of the spring corresponding to the maximum force should be approximately 30 mm. The spring index can be taken as 6. The spring is made of patented and cold-drawn steel wire. The ultimate tensile strength and modulus of rigidity of the spring material are 1090 and 81 370 N/mm2 respectively. The permissible shear stress for the spring wire should be taken as 50% of the ultimate tensile strength. Design the spring and calculate:

(i) wire diameter;

(ii) mean coil diameter;

(iii) number of active coils;

(iv) total number of coils;

(v) free length of the spring; and

(vi) pitch of the coil.

Draw a neat sketch of the spring showing various dimensions.

Solution

Given :

P = 1250 N

d = 30 mm C = 6

Sut = 1090 N/mm2

G = 81 370 N/mm2,

t = 0.5 Sut

Step I *Wire diameter*

The permissible shear stress is given by,

$$\tau = 0.5\, S_{ut} = 0.5(1090) = 545 \text{ N/mm}^2$$

From Eq. (10.7),

$$K = \frac{4C-1}{4C-4} + \frac{0.615}{C} = \frac{4(6)-1}{4(6)-4} + \frac{0.615}{6} = 1.2525$$

From Eq. (10.13),

$$\tau = K\left(\frac{8PC}{\pi d^2}\right) \quad \text{or} \quad 545 = (1.2525)\left\{\frac{8(1250)(6)}{\pi d^2}\right\}$$

$$\therefore \qquad d = 6.63 \text{ or } 7 \text{ mm} \qquad \text{(i)}$$

Step III *Number of active coils*

From Eq. (10.8),

$$\delta = \frac{8PD^3N}{Gd^4} \quad \text{or} \quad 30 = \frac{8(1250)(42)^3 N}{(81\,370)(7)^4}$$

$\therefore \qquad N = 7.91$ or 8 coils (iii)

Step IV *Total number of coils*

It is assumed that the spring has square and ground ends. The number of inactive coils is 2. Therefore,

$N_t = N + 2 = 8 + 2 = 10$ coils (iv)

Step V *Free length of spring*

The actual deflection of the spring is given by,

$$\delta = \frac{8PD^3N}{Gd^4} = \frac{8(1250)(42)^3(8)}{(81370)(7)^4} = 30.34\text{ mm}$$

solid length of spring $= N_t d = 10(7) = 70$ mm

It is assumed that there will be a gap of 1 mm between consecutive coils when the spring is subjected to the maximum force. The total number of coils is 10.

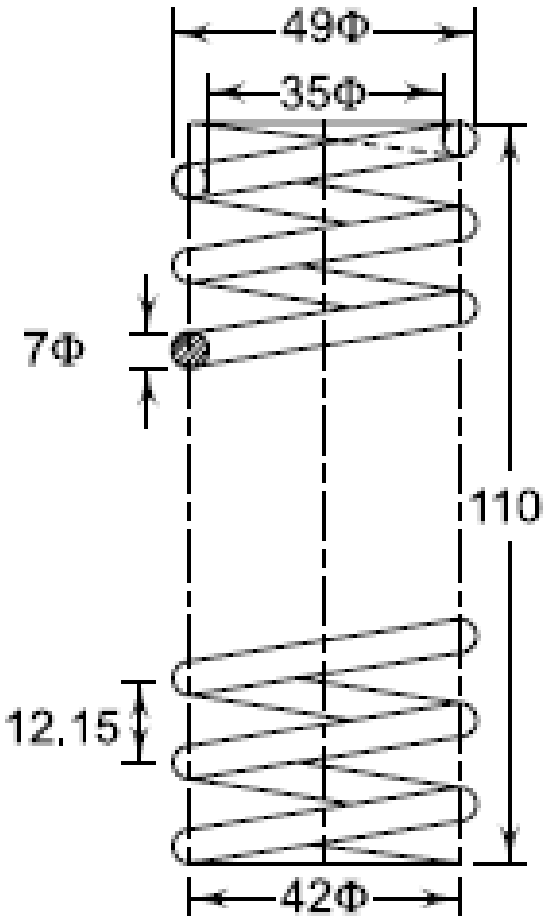
The total axial gap between the coils will be $(10-1) \times 1 = 9$ mm.

$$\text{Free length} = \text{Solid length} + \text{Total axial gap} + \delta$$
$$= 70 + 9 + 30.34$$
$$= 109.34 \text{ or } 110 \text{ mm} \qquad \text{(v)}$$

Step VI *Pitch of the coil*

$$\text{Pitch of coil} = \frac{\text{Free length}}{(N_t - 1)} = \frac{109.34}{(10 - 1)} = 12.15 \text{ mm} \qquad \text{(vi)}$$

The dimensions of the spring are shown in



DESIGN AGAINST FLUCTUATING LOAD

- In many applications, the force acting on the spring is not constant but varies in magnitude with time.
- The valve spring of an automotive engine is subjected to millions of stress cycles during its lifetime.
- On the other hand, the springs in linkages and mechanisms are subjected to comparatively less number of stress cycles.
- The springs subjected to fluctuating stresses are designed on the basis of two criteria—design for infinite life and design for finite life.
- Let us consider a spring subjected to an external fluctuating force, which changes its magnitude from

  *P*max. to *P*min. in the load cycle.

..

- The mean force ***Pm*** and the force amplitude *Pa* are given by

$$P_m = \frac{1}{2}(P_{\max.} + P_{\min.})$$

$$P_a = \frac{1}{2}(P_{\max.} - P_{\min.})$$

- The mean stress (tm) is calculated from mean force (Pm) by using shear stress correction factor (Ks). It is given by

$$\tau_m = K_s\left(\frac{8P_mD}{\pi d^3}\right)$$

$$K_s = \left(1 + \frac{0.5}{C}\right)$$

....

- *K*s is the correction factor for direct shear stress and it is applicable to mean stress only. For torsional stress amplitude (*ta*), it is necessary to also consider the effect of stress concentration due to curvature in addition to direct shear stress. Therefore,

$$\tau_a = K_s\ K_c \left(\frac{8P_a D}{\pi d^3} \right)$$

$$\tau_a = K \left(\frac{8P_a D}{\pi d^3} \right)$$

- For Patented and cold-drawn steel wires (Grade-1 to 4)

$$S'_{se} = 0.21\ S_{ut}$$

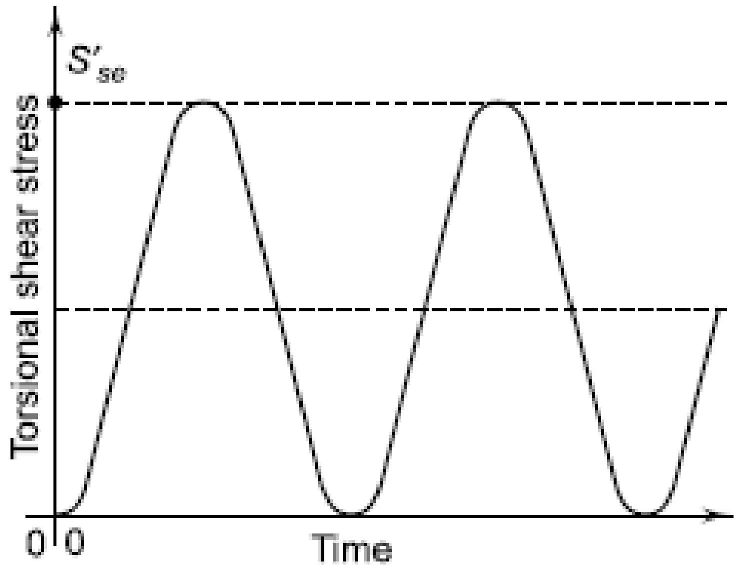
$$S_{sy} = 0.42\ S_{ut}$$

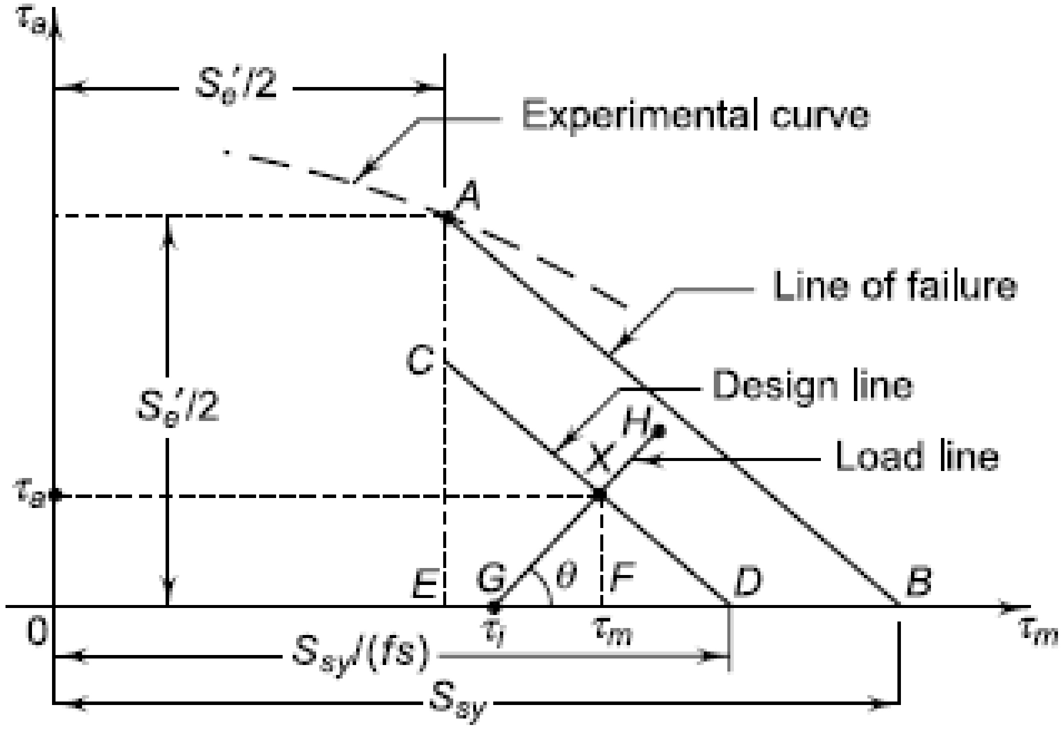
- For oil-hardened and tempered steel wires (SW) and VW grade),

$$S'_{se} = 0.22\ S_{ut}$$

$$S_{sy} = 0.45\ S_{ut}$$

where Sut is the ultimate tensile strength

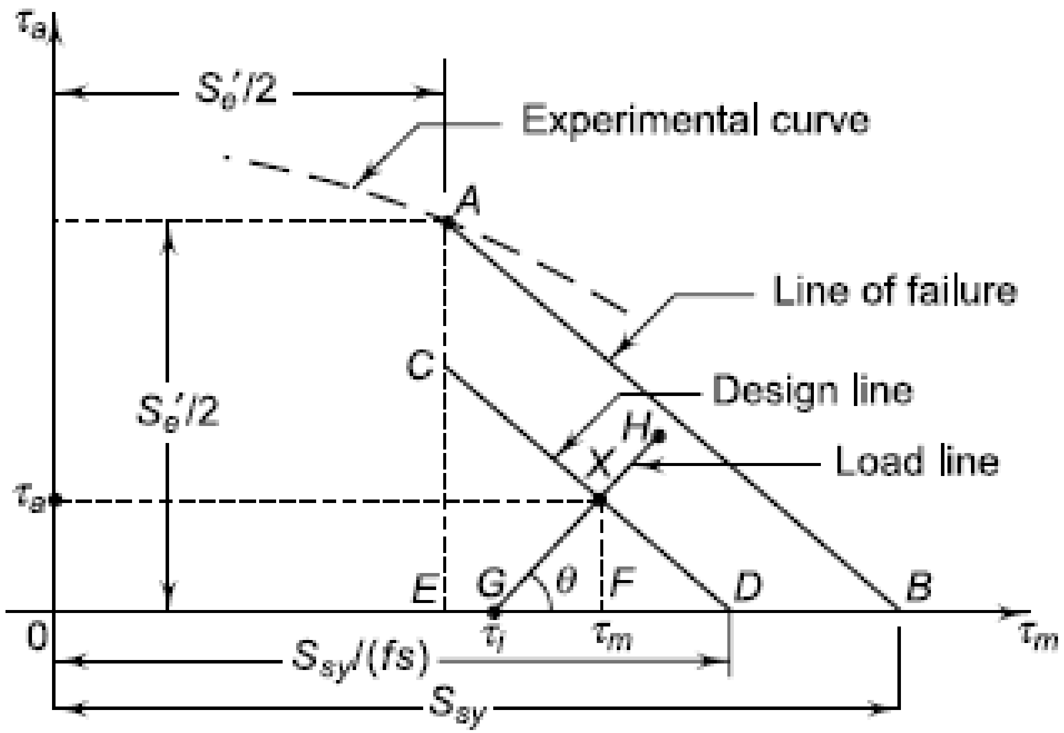




- To consider the effect of the factor of safety, a line *DC* is constructed from the point *D* on the abscissa in such a way that

$$\overline{OD} = \frac{S_{sy}}{(fs)}$$

- The line DC is parallel to the line BA.



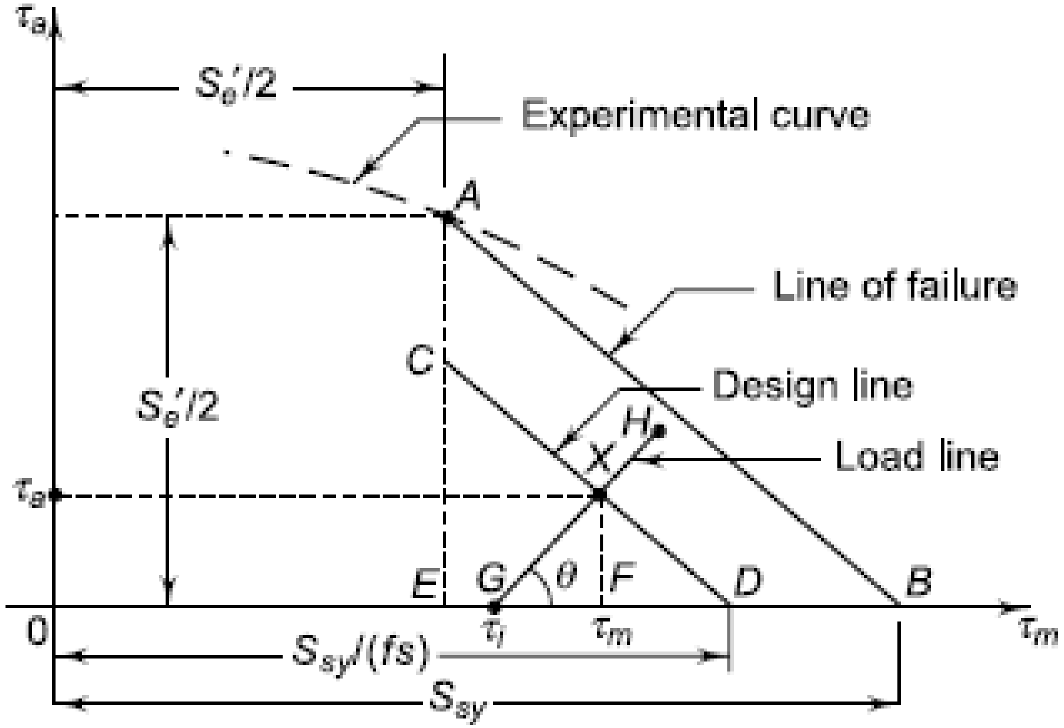
- Any point on the line CD, such as X, represents a stress situation with the same factor of safety.

- Line CD is called the design line because it is used to find out permissible stresses with a particular factor of safety.

- The line GH is called load line. It is drawn from the point G on the abscissa at a distance ti from the origin.

- The torsional shear stress due to initial pre-load on the spring (Pi) is ti. The line GH is constructed in such a way that its slope q is given by,

$$\tan\theta = \frac{\tau_a}{\tau_m}$$



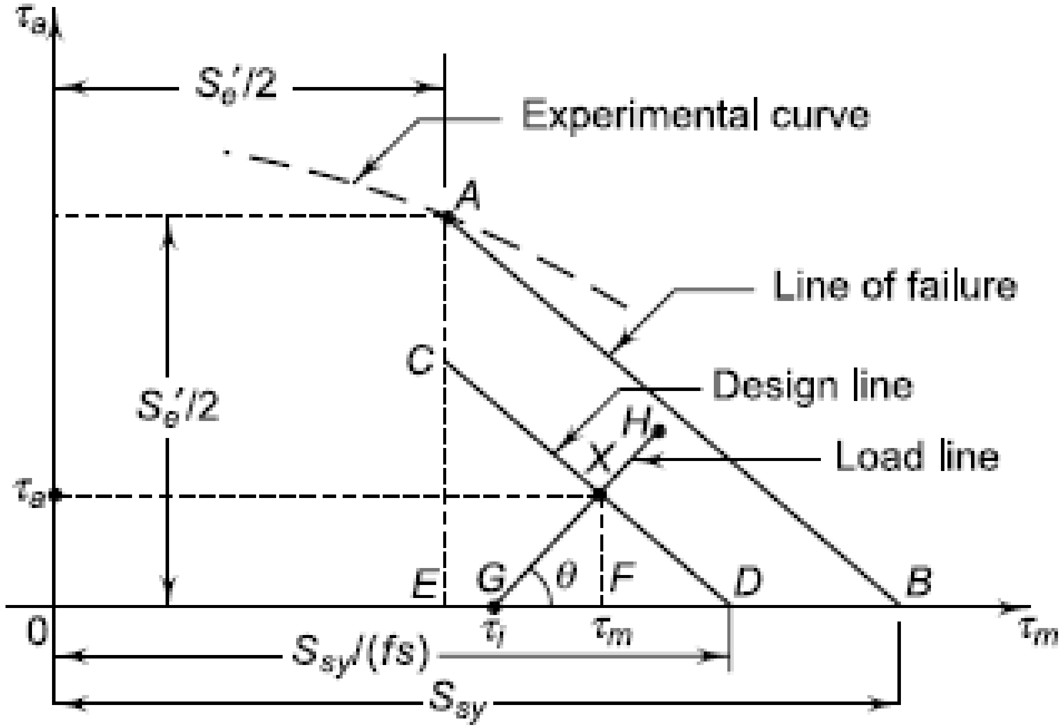
- The point of intersection between design line *DC* and load line *GH* is *X*.
- The coordinates of the point *X* are (t_m, t_a).

Considering similar triangles *XFD* and *AEB*,

$$\frac{\overline{XF}}{\overline{FD}} = \frac{\overline{AE}}{\overline{EB}}$$

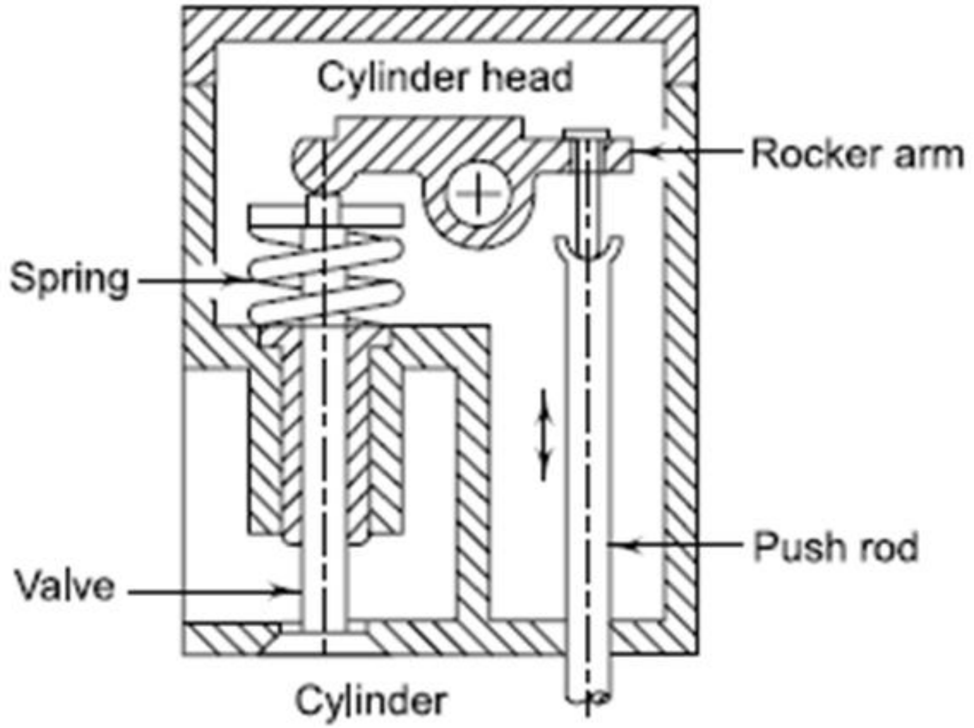
or
$$\frac{\tau_a}{\frac{S_{sy}}{(fs)} - \tau_m} = \frac{\frac{1}{2} S'_{se}}{S_{sy} - \frac{1}{2} S'_{se}} \qquad (10.22)$$

The above equation is used in the design of springs subjected to fluctuating stresses.



Problem

The constructional details of an exhaust valve of a diesel engine are shown in Fig. 10.22. The diameter of the valve is 32 mm and the suction pressure in the cylinder is 0.03 N/mm2. The mass of the valve is 50 g. The maximum valve lift is 10 mm. The stiffness of the spring for the valve is 10 N/mm. The spring index can be assumed as 8. The permissible shear stress in the spring wire is recommended as 30% of the ultimate tensile strength. Neglecting the effect of inertia forces, design the spring for static considerations and determine the factor of safety against fluctuating stresses.



Solution

Given

k = 10 N/mm

C = 8

t = 0.3 Sut

Step I *Maximum spring force*

The spring is subjected to fluctuating stresses. Therefore, oil-hardened and tempered valve spring wire of Grade-VW is selected for this application.

Initially, the spring is fitted with a pre-load. The initial pre-load should be sufficient to hold the valve on its seat against the negative pressure inside the cylinder during the suction stroke.

Since the cylinder is vertical, additional pre-load should be provided to account for the weight of the valve.

$$\text{Suction force} = \text{valve area} \times \text{suction pressure}$$

$$= \frac{\pi}{4}(32)^2(0.03) = 24.13 \text{ N}$$

$$\text{Weight of the valve} = mg = (0.05)(9.81) = 0.49 \text{ N}$$

$$\text{Minimum pre-load} = 24.13 + 0.49 = 24.62 \text{ N}$$

To be on the safer side, the initial pre-load is taken as 30 N.

During the exhaust stroke, the spring is further compressed by 10 mm (valve-lift).

The maximum force acting on the spring is given by

$$P\text{max.} = P\text{min.} + kd = 30 + 10(10) = 130 \text{ N}$$

...

Step II *Design against static load*

$$K = \frac{4C-1}{4C-4} + \frac{0.615}{C} = \frac{4\,(8)-1}{4\,(8)-4} + \frac{0.615}{8} = 1.184$$

$$K_s = 1 + \frac{0.5}{C} = 1 + \frac{0.5}{8} = 1.0625$$

From Eq. (10.13),

$$\tau = K\left(\frac{8PC}{\pi d^2}\right) \quad \text{or} \quad \tau = (1.184)\left[\frac{8(130)(8)}{\pi d^2}\right]$$

$$\text{or} \quad \tau = \frac{3135.63}{d^2}\ \text{N/mm}^2 \qquad \text{(a)}$$

- The permissible shear stress is denoted by *td* in order to differentiate it from the induced stress *t*. It is given by,

$$\tau_d = 0.3\ S_{ut}$$

Equations (a) and (b) are solved by the trial and error method.

Trial 1

$d = 2.5$ mm

$$\tau = \frac{3135.63}{d^2} = \frac{3135.63}{(2.5)^2} = 501.7\ \text{N/mm}^2$$

From Table 10.2,

$S_{ut} = 1470\ \text{N/mm}^2$

$\tau_d = 0.3\ S_{ut} = 0.3(1470) = 441\ \text{N/mm}^2$

Therefore, $\tau > \tau_d$

The design is not safe.

…

Trial 2

$d = 3$ mm

$$\tau = \frac{3135.63}{d^2} = \frac{3135.63}{(3)^2} = 348.4 \text{ N/mm}^2$$

From Table 10.2,

$S_{ut} = 1430$ N/mm^2

$\tau_d = 0.3\ S_{ut} = 0.3(1430) = 429$ N/mm^2

Therefore, $\tau < \tau_d$

- The design is satisfactory and the wire diameter should be 3 mm.
- However, the spring is subjected to fluctuating stresses and to account for these stresses, the wire diameter is increased to 4 mm.

$d = 4$ mm

$D = Cd = 8(4) = 32$ mm

From Eq. (10.9),

$$N = \frac{Gd^4}{8D^3k} = \frac{81370(4)^4}{8(32)^3(10)} = 7.95 \text{ or } 8 \text{ coils}$$

It is assumed that the spring has square and ground ends.

$$N_t = N + 2 = 8 + 2 = 10 \text{ coils}$$

From Eq. (10.8),

$$\delta = \frac{8PD^3N}{Gd^4} = \frac{8(130)(32)^3(8)}{(81370)(4)^4} = 13.09 \text{ mm}$$

…

- Solid length of spring = Ntd = 10(4) = 40 mm
- It is assumed that there will be a gap of 0.5 mm between consecutive coils when the spring is subjected to the maximum force.
- The total number of coils is 10. The total axial gap between the coils will be (10 – 1) \ 0.5 = 4.5 mm.
- Free length = solid length + total axial gap + d
- Free length = 40 + 4.5 + 13.09 = 57.59 or 60 mm

Step III *Factor of safety against fluctuating stresses*

$$P_m = \frac{1}{2}(P_{\max.} + P_{\min.}) = \frac{1}{2}(130 + 30) = 80 \text{ N}$$

$$P_a = \frac{1}{2}(P_{\max.} - P_{\min.}) = \frac{1}{2}(130 - 30) = 50 \text{ N}$$

From Eq. (10.18) and (10.19),

$$\tau_m = K_s\left(\frac{8P_mD}{\pi d^3}\right) = (1.0625)\left(\frac{8(80)(32)}{\pi(4)^3}\right)$$

$$= 108.23 \text{ N/mm}^2$$

$$\tau_a = K\left(\frac{8P_aD}{\pi d^3}\right) = (1.184)\left(\frac{8(50)(32)}{\pi(4)^3}\right)$$

$$= 75.38 \text{ N/mm}^2$$

From Table No. 10.2, ($d = 4$ mm)

$S_{ut} = 1400 \text{ N/mm}^2$

- From. Eq. (10.21), the relationships for oil hardened and tempered steel wire are as follows:

  S¢se = 0.22 *Sut* = 0.22(1400) = 308 N/mm2

  Ssy = 0.45 *Sut* = 0.45(1400) = 630 N/mm2

  From Eq. (10.22),

$$\frac{\tau_a}{\left(\frac{S_{sy}}{fs}\right) - \tau_m} = \frac{\frac{1}{2}S'_{se}}{S_{sy} - \frac{1}{2}S'_{se}}$$

or
$$\frac{75.38}{\left(\frac{630}{fs}\right) - 108.23} = \frac{\frac{1}{2}(308)}{630 - \frac{1}{2}(308)}$$

$$\frac{75.38}{\left(\frac{630}{fs}\right) - 108.23} = \frac{154}{476}$$

$$\frac{630}{(fs)} - 108.23 = \frac{75.38(476)}{154} = 232.99$$

$$\frac{630}{(fs)} = 108.23 + 232.99$$

$$(fs) = 1.85$$

The factor of safety against fluctuating stresses is reasonable.

- ***Step IV*** *Spring specifications*

  (i) Material = oil-hardened and tempered steel wire of Grade-VW

  (ii) Wire diameter = 4 mm

  (iii) Mean coil diameter = 32 mm

  (iv) Free length = 60 mm

  (v) Total number of coils = 10

  (vi) Style of ends = square and ground

End of Part II